BERKELEY LAB

LAWRENCE BERKELEY NATIONAL LABORATORY

U.S. DEPARTMENT OF ENERGY

Carbon Fiber Laminate Theory (Laminated Plate Theory)

LBNL Composites Workshop

February 29-March 3, 2016

Overview

What are composites and why we use them

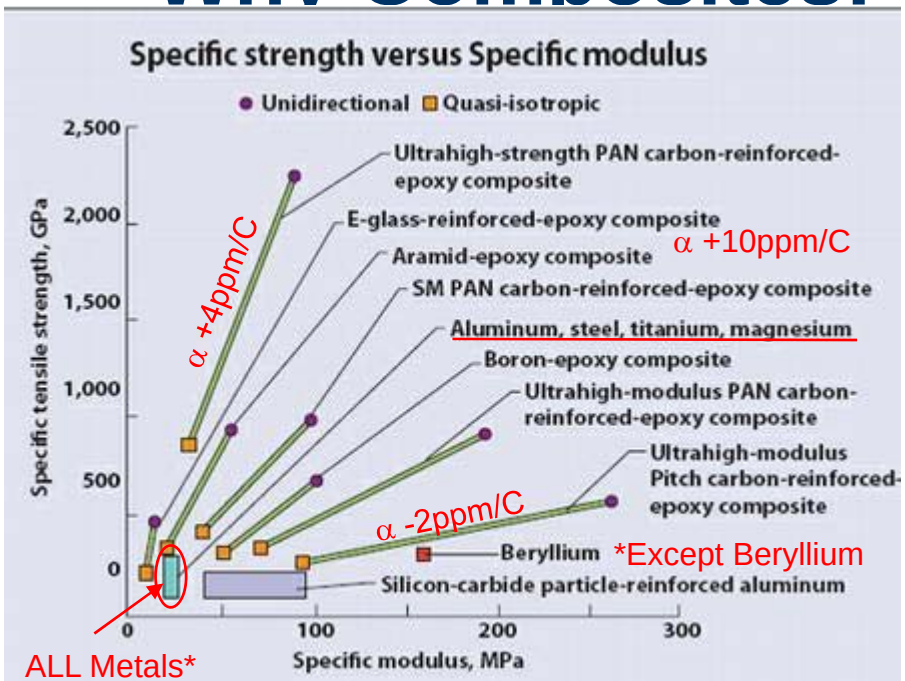
Very brief introduction to design estimation

Design resources (under construction)

Q&A

For this discussion: ‘Carbon Fiber’ as a material, is Carbon Fiber Reinforced Plastic (CFRP)—a ‘Composite’. Composites with other fibers and matrices are also broadly mentioned.

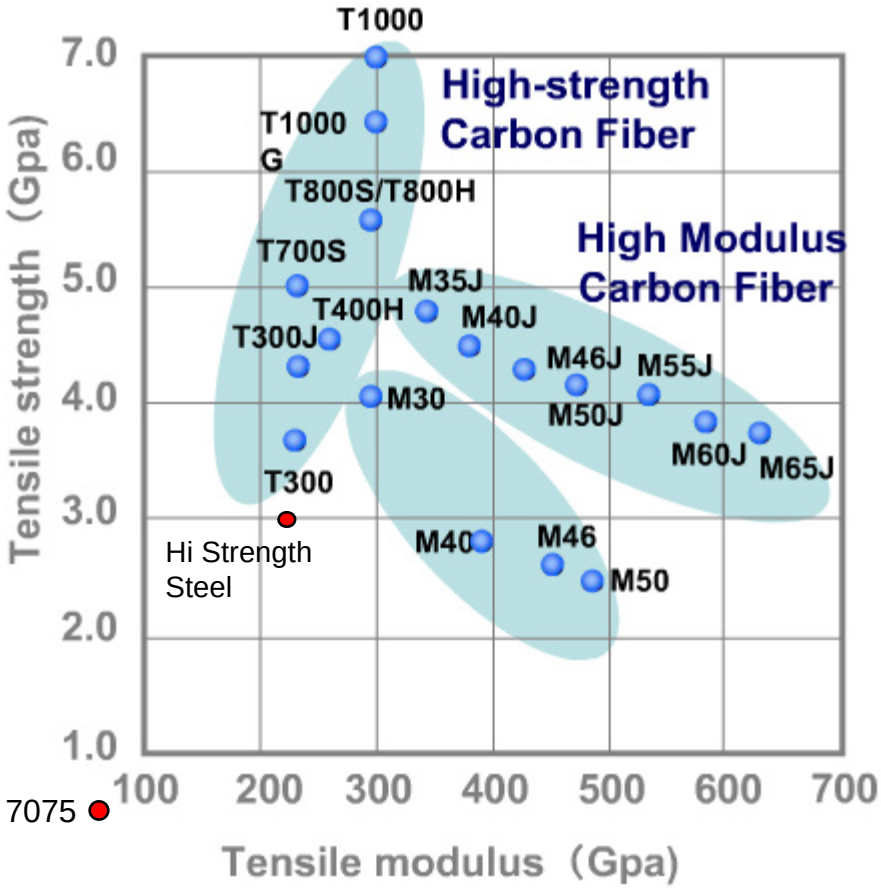
Why Composites: It's not just mass



In units of E/ρ where Aluminum = 1 CFRP has a ρ ~60% of Al, GFRP similar; Modulus (E) of CFRP tunable from just under Aluminum to just over Titanium with Quasi-isotropic laminates. Oriented laminates can exceed Berylium in a desired direction.

- Clearly where mass is critical composite materials excel!
- Main competition is Be—not desired for several reasons
- Non-magnetic properties important in B-Field applications
- Glass fiber composites non-conductive for Hi-Voltage
- Thermal expansion tunable from near zero to Steel
- Matrix resin choice flexible based on other requirements

Carbon Fiber Properties



PAN (Poly Acrylo-Nitrile) is a polymer: (viscose). Pitch is geologic tar, a byproduct of oil extraction with very high carbon content and long polymer chains thus char ratio

- Carbon Fiber is not a material, it is a family of materials
- Other fibers such as Glass, PE, Kevlar, etc have more unique properties (tightly defined)
- CF properties vary based on their %-Graphitization—from mostly ‘glassy’ to ‘crystalline’
- Also on ‘precursor’ material e.g. PAN versus Pitch
- Chart is for PAN based fibers
- Pitch based fibers can have modulus in excess of 1000GPa

A Brief word on ‘Strength’

We tend to use composites in deflection driven designs, thus tend to use ultra-high modulus fibers

These fibers have low failure strains e.g. 0.3%

‘High Strength’ fibers have failure strains in excess of 1%

Matrices range from 1-5% failure strains

Strength models do exist to combine these in laminates, but require testing to use (Tsia-Hill or Tsai-Wu)—strain energy based like Von Mises…

For stiffness based designs, laminate strength is rarely an issue, but should be checked

Strength of composite materials will not be presented formally, but discussion is welcome

Consider using Fiber Strain as a metric to assess margins of safety

This technique is often referred to as ‘First Ply Failure’ and is rather conservate…

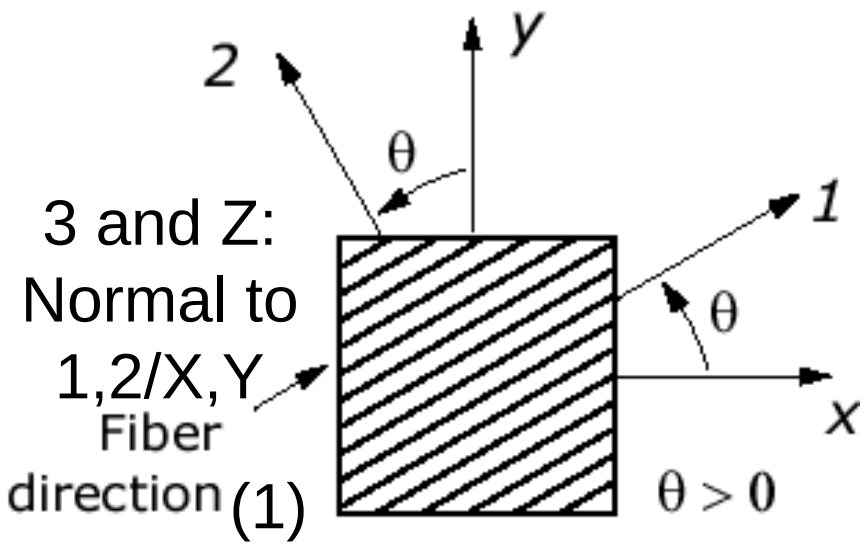
ACP Will report these values specifically

If you are using ANSYS without a composites package, do not use Von-Mises stresses…

Reporting principle strain of an isotropic solid is a quick estimate, but not proper.

Ultimately a more proper, full orthotropic analysis in ANSYS is required

Lamination: Additive manufacture

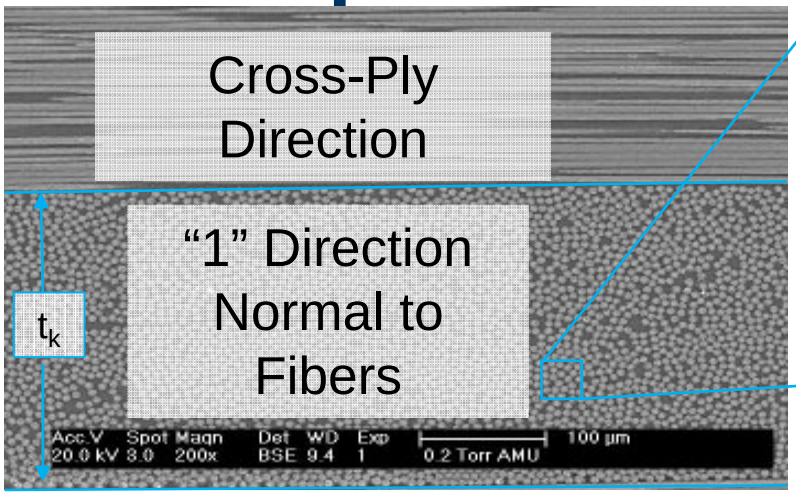


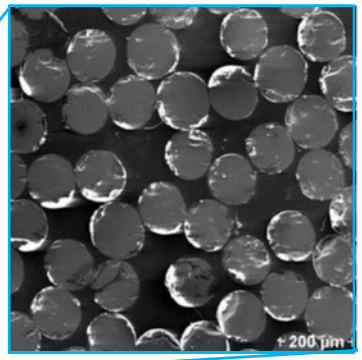
‘Carbon Fiber’ material is built layer by layer on a mold

Each layer has a fiber direction and ‘thickness’ specified by the design requirements (X,Y are Body Coordinates)

‘Thickness’ is a function of Fiber Areal Weight and Resin content—typically 55-60% Fiber Volume Fraction

Composite Material 'Lever Rule'





Modulus of 'unit' section:

$$E_c = E_f * V_f + E_m * V_m + E_v * V_v$$

Where:

Void Fraction

$$V_f + V_m + V_v = 1 \text{ (Unity)}$$

$E_c * t_k$ = Stiffness of lamina

Volume fractions of Fiber, Matrix, and Void content map directly to 'lamina' engineering properties

Various 'Volume Fractions' are a combination of material specification (FAW) and lamination process control

Lamina are layers in a 'laminate' LPT is used to predict the structural performance of a sum of lamina

Cured Ply Thickness (CPT = t_k)

Lamina Properties: Lever Rule in 2D

Lamina Properties

$\boxed{\text{Cloth Compliance} := 13\%}$

V_f = Fiber Volume Fraction ~ 0.50-0.60 depending on process

V_m = Matrix Volume Fraction ~1-V_f

Modulus Fiber (1) Direction

$$E_{11} := \frac{(E_{1f} \cdot V_f + E_m \cdot V_l)}{1 + \text{Cloth Compliance}}$$

$$G_{12} := \frac{G_m}{1 - V_f \cdot \left(1 - \frac{G_m}{G_{12f}}\right)}$$

$$\nu_{12} := \nu_{12f} \cdot V_f + \nu_m \cdot V_l$$

Modulus Orthogonal 2/3 Direction

$$E_{22} := \frac{E_m}{1 - V_f \cdot \left(1 - \frac{E_m}{E_{2f}}\right)}$$

$$G_{23} := \frac{G_m}{1 - V_f \cdot \left(1 - \frac{G_m}{G_{23f}}\right)}$$

$$\nu_{23} := \frac{E_{22}}{2 \cdot G_{23}} - 1$$

$$G_{13} := G_{12}$$

$$\nu_{13} := \nu_{12}$$

$$\nu_{21} := \nu_{12} \cdot \frac{E_{22}}{E_{11}}$$

$$[A] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

~E_{fiber}* V_f Modified by Cloth Compliance

~E_{matrix}* V_m

Actually C_{66}

Note: $(1+\nu_{12}\nu_{21})$ ~1.06-1.1

[C] represents *orthotropic* properties of one layer of composite material in 'Fiber Coordinates' (note not 6X6 due to orthotropic approximation)

"1" is fiber Direction, "2" is transvers to fiber (in plane), "3" is thru thickness dimension (ignored for orthotropic case)

Matrix moduli ~1-2 orders of magnitude less than fiber

ν_{12f} (fiber poisson ratio) not well published, ranges from 0.2-0.35

'Cloth Compliance' is empirical—not all fiber contributes to in-plane properties ranges from 8-15% base on weave ("0" for Uni-Directional Tape)

Lamina properties ~50% Fiber Properties in modulus, even less with cloth

Not all micromechanics use the same equations

Chou

Lamina Properties

$\text{Cloth_Compliance} := 0\%$

Modulus Fiber (1) Direction

$$E_{11} := \frac{(E_{1f} \cdot V_f + E_m \cdot V_l)}{1 + \text{Cloth_Compliance}}$$

Modulus Orthogonal 2/3 Direction

$$E_{22} := \frac{E_m}{1 - V_f \cdot \left(1 - \frac{E_m}{E_{2f}}\right)}$$

$$G_{12} := \frac{G_m}{1 - V_f \cdot \left(1 - \frac{G_m}{G_{12f}}\right)} \qquad G_{23} := \frac{G_m}{1 - V_f \cdot \left(1 - \frac{G_m}{G_{23f}}\right)} \qquad G_{13} := G_{12}$$

$$\nu_{12} := \nu_{12f} \cdot V_f + \nu_m \cdot V_l \qquad \nu_{23} := \frac{E_{22}}{2 \cdot G_{23}} - 1 \qquad \nu_{13} := \nu_{12} \qquad \nu_{21} := \nu_{12} \cdot \frac{E_{22}}{E_{11}}$$

Kollar-Springer

11.3 NOTE ON THE MICROMECHANICS MODELS

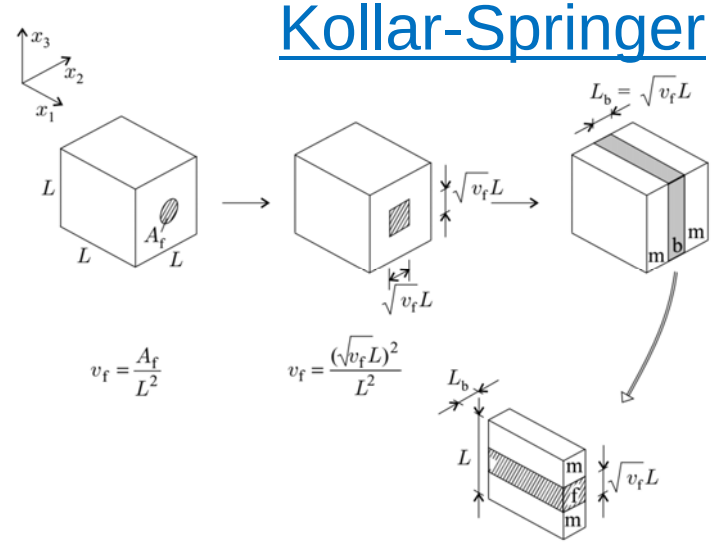


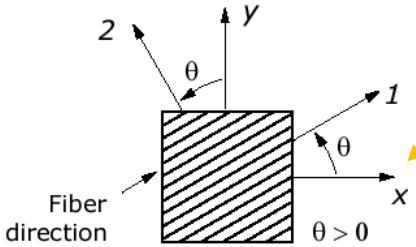
Figure 11.7: The model for the modified rule of mixtures (top) and the representation of the middle "b" layer (bottom).

$$E_{b2} = \sqrt{v_f} E_{f2} + (1 - \sqrt{v_f}) E_m.$$

Different methods to calculate E22 (transverse lamina modulus)

Differ by ~6% but effect is small—E11>>E22

Oriented Lamina Properties: Transform



(Properties defined by [C] in fiber coordinates)

Primary Body Direction (X)

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

(Coordinate Transform Matrix)

$$[C_{body}] = [T]^{-1}[C][T]^T$$

$[T]^T$ (transpose) works out due to definition of [C] and mapping to 'engineering strain' via [R] factor 2 In G_{12} term

Sensitivity

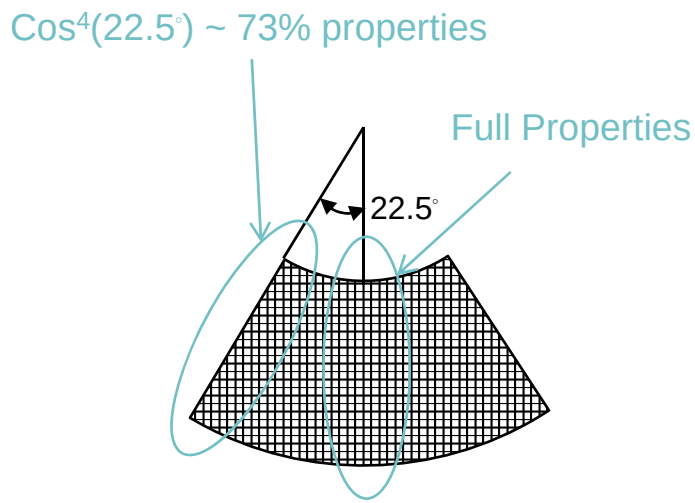
Common Orientations

Transverse Direction

Theta (Degrees)	Cos^4(Theta) (Value)
5	0.984
10	0.941
15	0.840
45	0.250
60	0.063
90	0.000 (matrix)

- **Transform to 'Body Coordinates' (X, Y) is ~ $\cos^4(\theta)$**
- **X is primary direction, Y is transverse**
- **Body Coordinates are aligned with physical structure, convention is $\theta = 0$ is aligned with X**
- **Sensitivity shows accuracy required during lamination and/or design**

Properties dependent on orientation



Local analysis coordinate systems are important for material definition in ANSYS (FEM)

Flat plates and Cylinders are simple—body and fiber coordinates retain a unique mapping throughout the structure

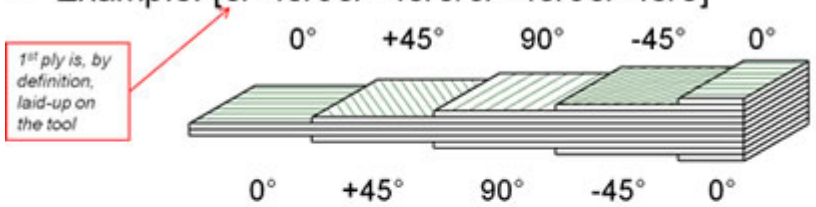
Transition elements: flanges or other non-planar structures are more complicated—fiber orientation thru volume needs to be accounted for

"Laminate" properties are an average over multiple layers

This can be done by hand calculation but software exists to aid in this

Laminate Properties: Multiple Lamina

- Laminates are often described by an orientation code
- Example: [0/-45/90/+45/0/0/+45/90/-45/0]



- Short hand: $[0/-45/90/+45]_S$ — symmetric
- Short hand: $[0/-45/90/+45]_N$ — Repeat N times
- Other Examples: $[0/+45/-45/90]_{2S}$ — 16 total plies
  $[0_2/+45/-45/90_2]_S$ — 12 total plies
  $[0_U/+45_{CL}]$ — 2 total plies (1 uni @ 0° and 1 cloth @ ±45°)

http://www.quartus.com/resources/white-papers/composites-101/

The above laminate is QIBS
"Quasi-Isotropic Balanced Symmetric"
'Quasi-Isotropic' modulus in plane
'Balanced' about the mid-plane by area
'Symmetric' matched orientations about mid-plane
$[0,60,-60]_S$ and $[0,90]_S$ are also QIBS

Laminate nomenclature describes orientations of layers

Generally assumes all layers are the same material

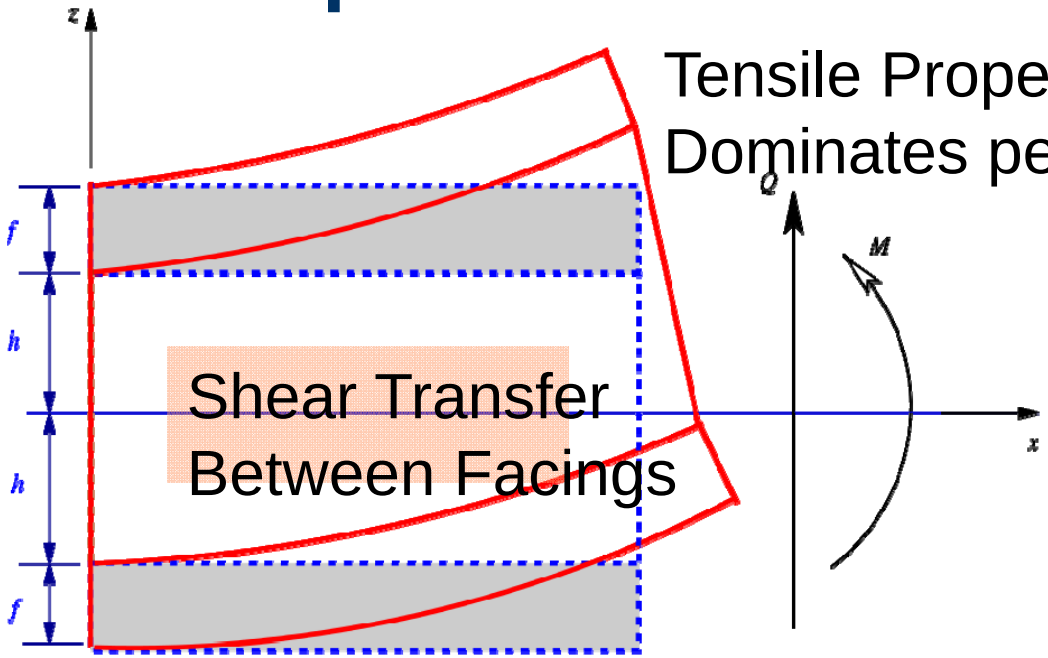
Shorthand is not applicable for 'hybrid' materials e.g. materials with different fiber/thickness

Later analysis assumes common material per layer/ply—generalization to hybrid materials is straightforward…

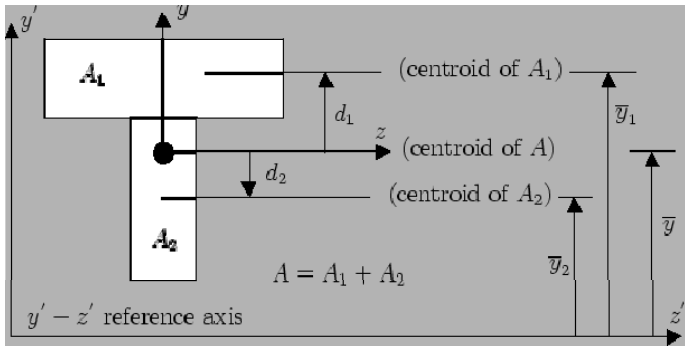
Sum of stiffness contributions of each layer: [A] matrix divided by thickness is the Modulus of the laminate

This is good for preliminary design

Composite Beam Theory



Tensile Properties of Facings
Dominates performance



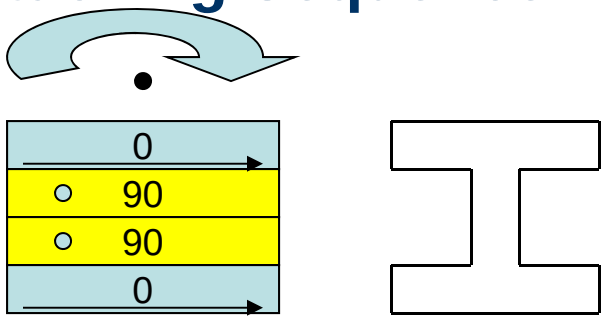
Composite Beam Theory is a method for adding various sections to calculate bending stiffness (weighted by offset from 'neutral axis')

Sectional inertia is also weighted by the stiffness of each section e.g. an Aluminum A_1 versus a Steel A_2 in the example above

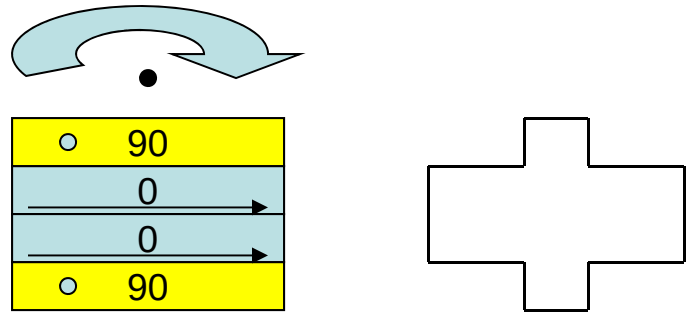
LPT is an identical formality—it simply sums smaller elements to arrive at similar section properties for tension, shear, and bending.

The matrix formalism of LPT is simply an accounting mechanism…

Stacking Sequence



Looks Like I-Beam when bent



Looks **NOT** Like I-Beam when bent on this axis

A Symmetric Laminate is symmetric wrt to ply orientation above and below the laminate mid-plane

- Example [0,+30,+30,0] sometimes written $(0,+30)_s$ is symmetric but not balanced

Balanced laminate is one where for every $+\theta$ there is also a $-\theta$ lamina

- Example [0, +30, -30, -30, +30, 0] or $[0, +30, -30]_s$

For a symmetric laminate, [B] = 0 always

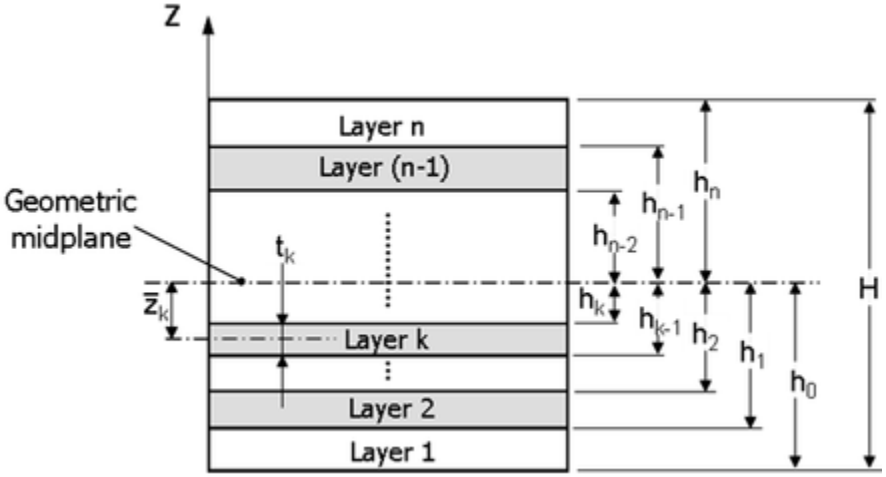
For Balanced laminates, $A_{16} = A_{26} = 0$ i.e. no shear extension coupling

Stacking sequence does not affect [A] matrix

- Both Laminates above have same Tensile properties—same [A] matrix

Laminate Plate Theory (nutshell)

Note: calculated from mid-plane
NOT neutral axis…



Δh_k (ply thickness t_k of each ply) is constant here—unimportant for [A], important for [B], and [D].

$$A_{ij} = \sum_{k=1}^{N} (\bar{C}_{ij})_k (z_k - z_{k-1}) = \sum_{k=1}^{N} (\bar{C}_{ij})_k t_k$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\bar{C}_{ij})_k (z_k^2 - z_{k-1}^2) = \sum_{k=1}^{N} (\bar{C}_{ij})_k t_k \bar{z}_k$$

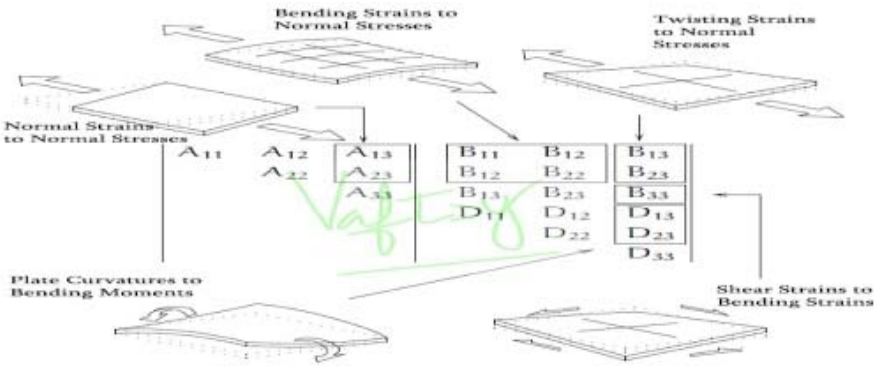
$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\bar{C}_{ij})_k (z_k^3 - z_{k-1}^3) = \sum_{k=1}^{N} (\bar{C}_{ij})_k \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right)$$

http://cae.vaftsycae.com/abd_matrix_composites.html

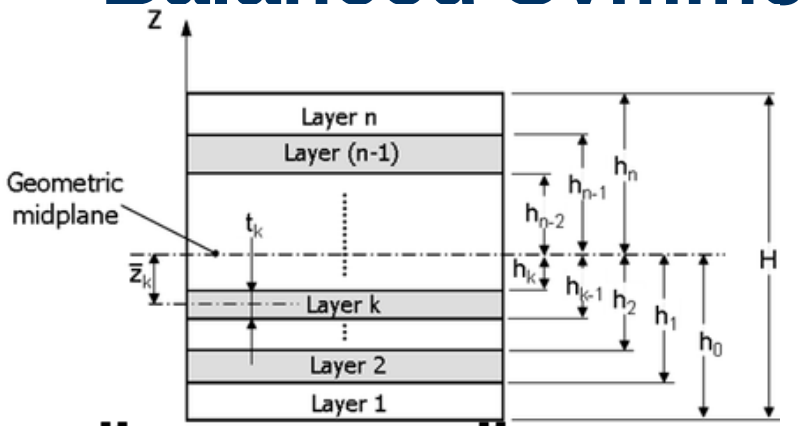
LPT has a matrix formalism which seems overtly complex:

- The ‘A’ matrix: Tensile Prop’s
- The ‘B’ matrix: Shear Coupling
- The ‘D’ Matrix: Plate Bending
- These are all about the mid-plane of the laminate (not section)

‘A’ matrix dominates for most ‘beam-like’ structures!



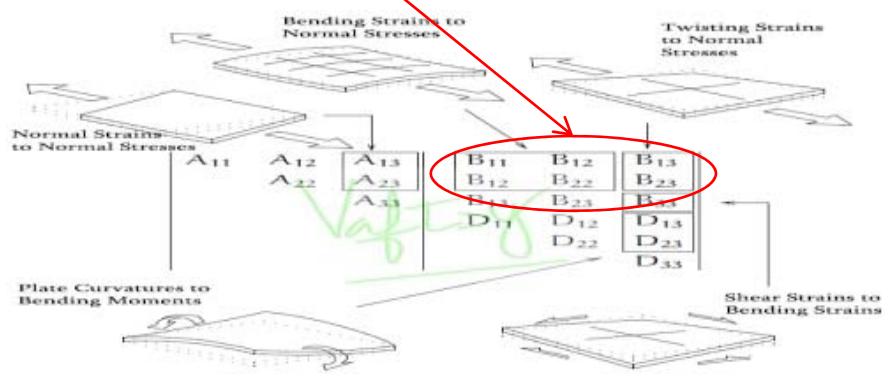
Balanced Symmetric laminates: no [B]



$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N}(\bar{C}_{ij})_k\left(z_k^2 - z_{k-1}^2\right) - \sum_{k=1}^{N}(\bar{C}_{ij})_k t_k \bar{z}_k$$

t_k is signed:
Negative below
Geometric mid-plane
Positive above

Ignore for Balanced Symmetric



Balanced Symmetric Identically cancels all [B] matrix elements!

[B] is un-fun to deal with without specific expertise

Balanced Symmetric laminates render [A] and [D] essentially independent

B and D are second, even third, order problems for most structures—they mostly come into consideration for 'local' loading of structures

[A] (tensile properties) dominate for most applications

[A] Matrix properties (Tensile Properties)

Theta (Degrees)	Cos^4(Theta) (Value)
0	1
60	0.063
-60	0.063
-60	0.063
60	0.063
0	1
Average:	0.375

6-ply QIBS Laminate

8-ply QIBS Laminate

Theta (Degrees)	Cos^4(Theta) (Value)
0	1
45	0.250
90	0
-45	0.250
-45	0.250
90	0
45	0.250
0	1
Average:	0.375

Average assumes all layers are equal thickness thus stiffness in the laminate coordinates

Both examples are QIBS: *ALL* examples of QIBS will have the same body modulus

Tables indicate orientation knockdown—still need to include Fiber Volume Fraction (V_f) knockdown

Tables above show fractional contributions of each layer in "X" direction (body coordinate) based on orientation "θ" of fibers to body

Each layer should also be knocked down by Vf ~50-60%

Including V_f, E_X and E_Y for a QIBS laminate range from 18-22% of E_{fiber}

QIBS laminates can be estimated as 'black' metal with some caveats

‘Black Aluminum’ and other estimates

‘High Strength’ CF ranges from 220-300GPa

‘Intermediate Modulus’ CF from 280-500GPa

Pitch based fibers start at 500 and goes to ~1000GPa

Considering the QIBS estimate of ~20% fiber modulus there exists both an Al and Ti equivalent quite trivially

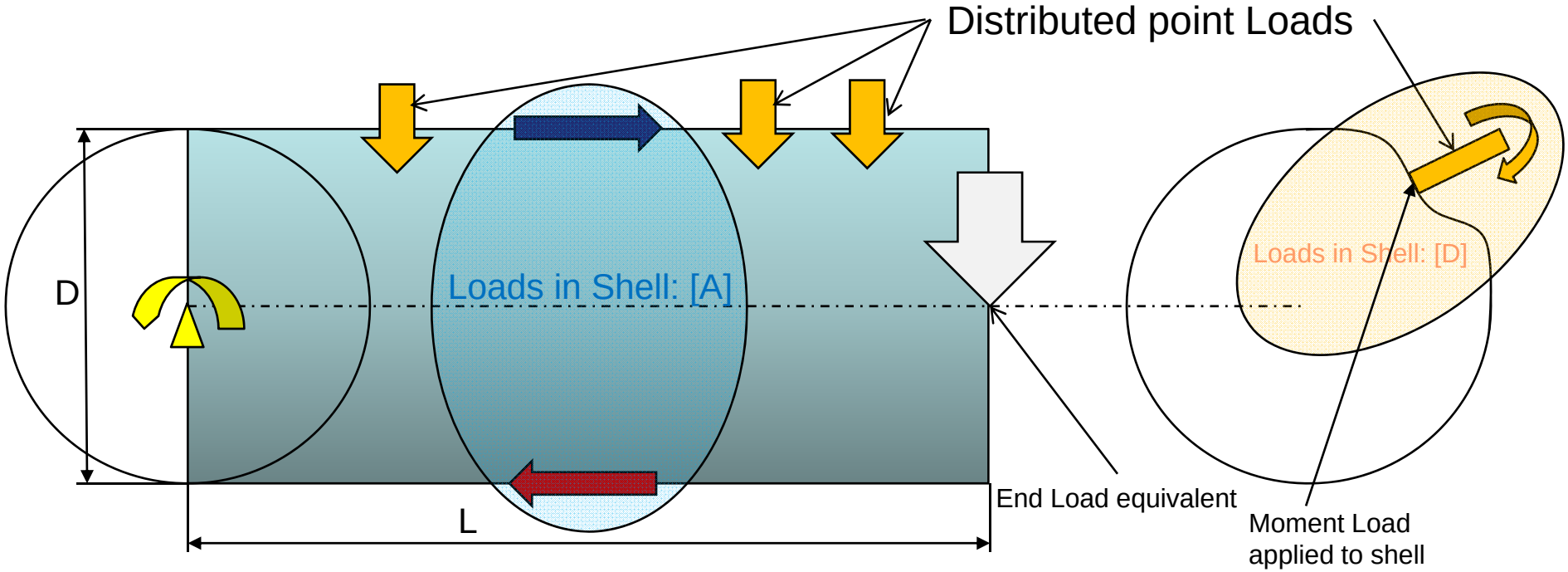
‘Black Aluminum’ is a pejorative term aimed at a design that took no other advantage other than density by using a CFRP component (since the early ‘80’s)

On the other hand, using an Aluminum or Titanium analog (with lower density) in analysis is a quick way to asses if a composite structure is beneficial in an FEM

As with any mechanical assembly, it’s usually the joints and local loads that screw with the design…

BERKELEY LAB

Global versus Local Deflections



Deflections may be dominated by local flexure for thin walled structures

L / D > 8 is required for ‘beam like’ behavior; else: ‘shear’ properties dominate global deflection relative to supports

Introducing loads into thin shells requires some expertise

Expert advice: when it’s needed

Use of ‘Black’ isotropic analogs are useful for design studies

- Approximations are truly valid for tensile (in-plane) loads
- Conceptual design studies, nominal sizing, first order mass…
- Feature or load rich locales are where approximations break

Normal loads/local moments need expertise to asses

- Localized load transfer into shell is important to understand

Joint compliance is significant for bolted/mechanical joints

- More than expected compared to metallic grips, however metallic joints are not frequently modelled properly…

Composites cannot always replace metallic solutions

An intermediate goal is to disseminate what’s easy to do, but also what’s hard…

Composites Engineering at LBNL

As with many disciplines at the lab; expert resources are matrixed, but available

Similarly, Composite Design is not broadly taught in an engineering curriculum (more in the past decade)

Cryogenics and Vacuum technology are similar examples:

- Engineering and Technical staff new to the lab become proficient quickly thru exposure
- Training is available both off and on-site
- Some problems still require an expert to solve—knowing who to talk to is important onsite and within the industry

Eng Div is looking to put together some seminars at various technical levels to teach Composite Design/Fab

BERKELEY LAB

Conclusion

The material is intended to give an idea of whether composites are useful for a design

Also, with the limitation of when to seek experts

Hand Calculations will get you rather close, but ultimately detailed FEA is required to ‘get to the next level’

Resources are available at LBNL, CERN

Design Resources

ECAnderssen@lbl.gov
JHSilber@lbl.gov
NDHartman@lbl.gov

Presently internal resource at LBNL—looking to share outside—will post most resources to this indico page

https://composites.lbl.gov (requires LDAP login)