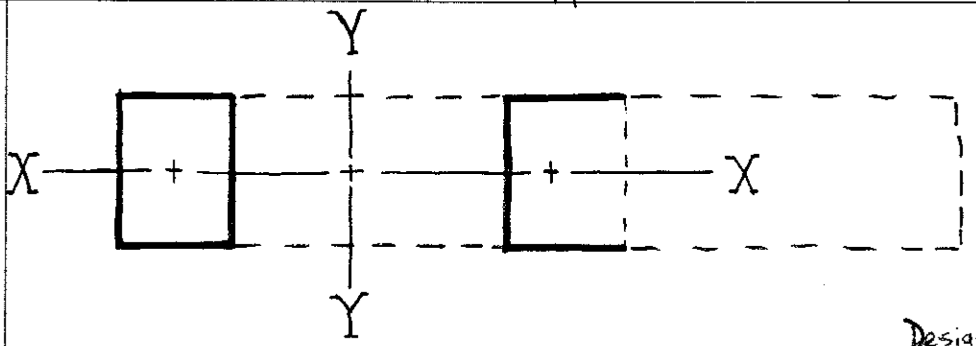
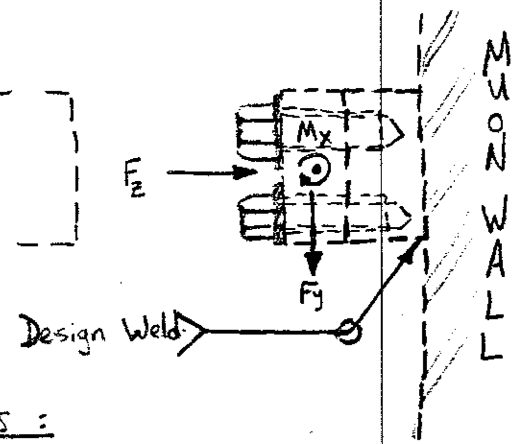
WELD CALCULATIONS

Using *Steel Structures, Design and Behavior*, 3rd edition, the welds that attach the support arms to the muon wall and then in-turn support the main T-beam were calculated and are attached in this section. Also, copies of the textbook pages on this subject are enclosed.

Using the LRFD (Load Resistance Factor Design) the required weld size was calculated to be a 9/32 inch fillet weld. However, AISC, *Manual of Steel Construction*, 9th edition specifies a minimum fillet weld of 5/16 inch. Therefore, an E70, full penetrating, fillet weld of size 5/16 inch will be used.





Combined Moments of Both Weld Groups:

$I_{x_{Total}} = 168.31 \text{ in}^4$ $\qquad C_x = 3.5 \text{ in}$

$I_{y_{Total}} = 534.28 \text{ in}^4$ $\qquad C_y = 7.203 \text{ in}$

$I_{Polar} = I_x + I_y = 702.59 \text{ in}^4$

Transform Weld Group Properties into LINE WELD Properties:

$$S_x = \frac{I_x}{c_x\, t} = \frac{(168.31 \text{ in}^4)}{(3.5\text{in})(1 \text{ in})} = 48.09 \text{ in}^2$$

$$S_y = \frac{I_y}{c_y\, t} = \frac{534.28 \text{ in}^4}{(7.203\text{in})(1\text{in})} = 74.17 \text{ in}^2$$

$$I_{Polar} = \frac{I_{polar}}{t} = \frac{702.59 \text{ in}^4}{1\text{in}} = 702.59 \text{ in}^3$$

Define Forces & Moments:

$F_x = \emptyset$

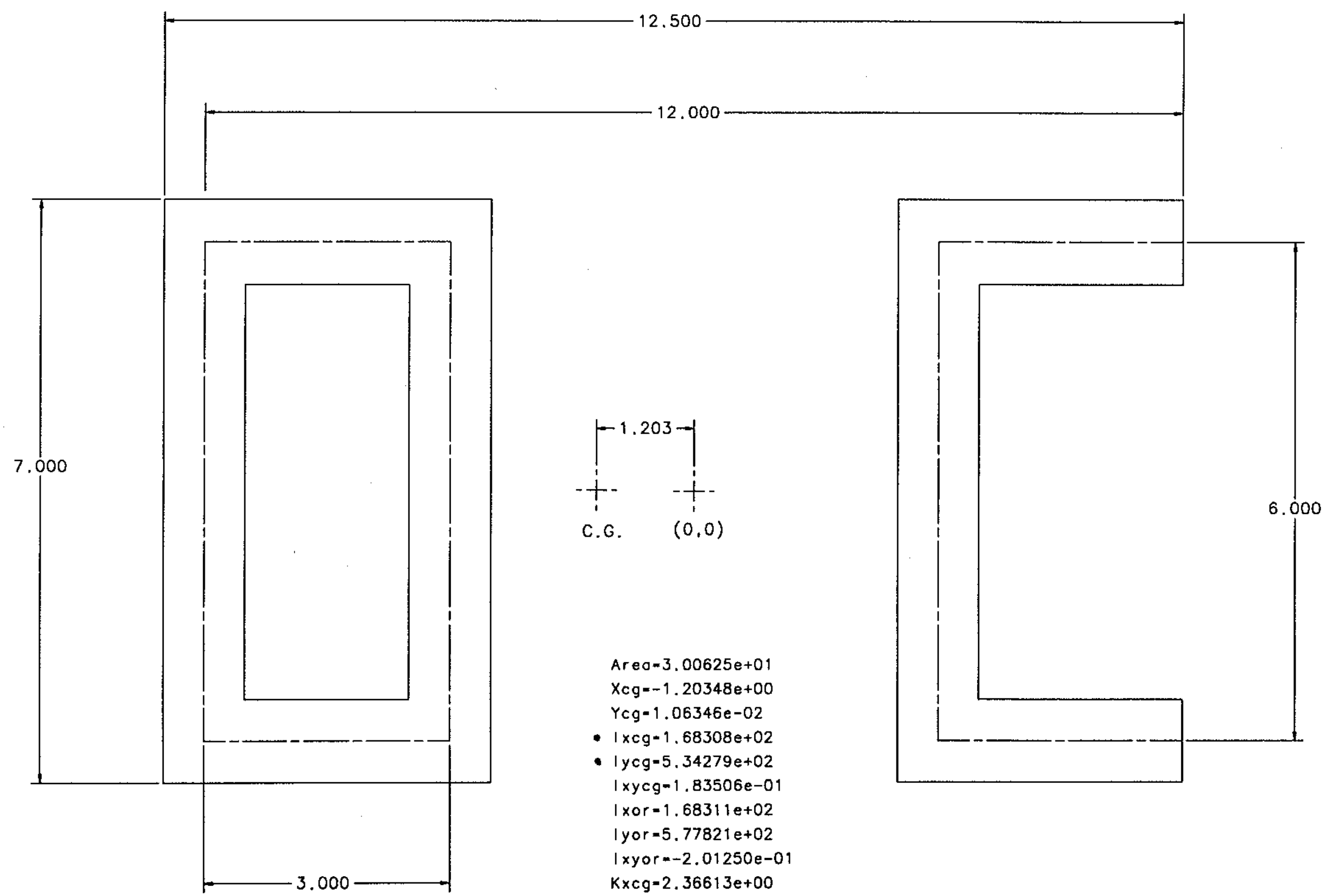
$F_y = 4215 \text{ lbs}$

$F_z = 6718 \text{ lbs}$

$M_x = 107{,}800 \text{ in-lb}$

$M_y = (F_z)\, 19.203\text{in} = (6718 \text{ lbs})(19.203 \text{ in}) = 129{,}006 \text{ in-lb}$

Due to close proximity, assume that the forces and moments act in the plane of the weld group.



Area=3.00625e+01
Xcg=-1.20348e+00
Ycg=1.06346e-02
• Ixcg=1.68308e+02
• Iycg=5.34279e+02
Ixycg=1.83506e-01
Ixor=1.68311e+02
Iyor=5.77821e+02
Ixyor=-2.01250e-01
Kxcg=2.36613e+00
Kycg=4.21572e+00
Kxor=2.36616e+00
Kyor=4.38414e+00
Ixprin=1.68307e+02
Iyprin=5.34279e+02
• Ipolar=7.02587e+02

Use 1" thick weld for unity in design calculations. They will be factored out later

Compute the Components of Force:

From direct shear:

$$R_{v_y} = \frac{4215 \text{ lbs}}{30 \text{ in}} = 140.5 \text{ \#/in}$$

From the Torsion "T" about the centroid of the Configuration:

$$R_z = \frac{Ty}{I_p} = \frac{(4215\#)(19.203 \text{ in})(3 \text{ in})}{(702.59 \text{ in}^3)} = 345.6 \text{ \#/in}$$

$$R_y = \frac{Tz}{I_p} = \frac{(4215\#)(19.203 \text{ in})(6)}{702.59 \text{ in}^3} = 691.2 \text{ \#/in}$$

From the Bending Moment about the Y-axis of the configuration:

$$R_{m_y} = \frac{M_y}{S_y} = \frac{129{,}006 \text{ in-lb}}{74.17 \text{ in}^2} = 1{,}739.3 \text{ lb/in}$$

From the shear in the Z-axis of the Configuration:

$$R_{V_z} = \frac{F_z}{L} = \frac{6718\#}{30 \text{ in}} = 223.93 \text{ in-lb}$$

From Torsion of the 3"x6"x24" beam exerted on the Top & Bottom Welds:

Torsion of Beam $= M_x = 107{,}800$ in-lb

$$\text{Coupled force} = \frac{107{,}800 \text{ in-lb}}{6 \text{ in}} = 17{,}967 \text{ in-lb.}$$

Shear force due to torsion:

$$\frac{17{,}967\#}{6''} = 2995 \text{ \#/in}$$

Calculate the Resultant Road:

$$R = \sqrt{(140.5 + 691.2)^2 + (345.6)^2 + (1739.3 + 223.93 + 2995)^2}$$

$$R = 5{,}039.37 \text{ lb/in}$$

According to the LRFD (Load Resistance Factor Design) the factored load (P_u) must be computed:

Dead load = 24.6 %   Live Load = 75.4 %

$$P_u = 1.2D + 1.6L = 1.2(.246)(4215\#) + 1.6(.754)(4215\#)$$

$$P_u = 6{,}329\,\#$$

The maximum load (#/in) on the weld due to the factored "P" is:

$$R_u = 5{,}039\left(6329/4215\right) = 7{,}566.27\ \#/in$$

Weld Resistance ΦR_{nw}:

$$\Phi R_{nw} = .75a(.707)(.60F_{EXX})$$ {where F_{EXX} is the strength of the steel not of the weld}

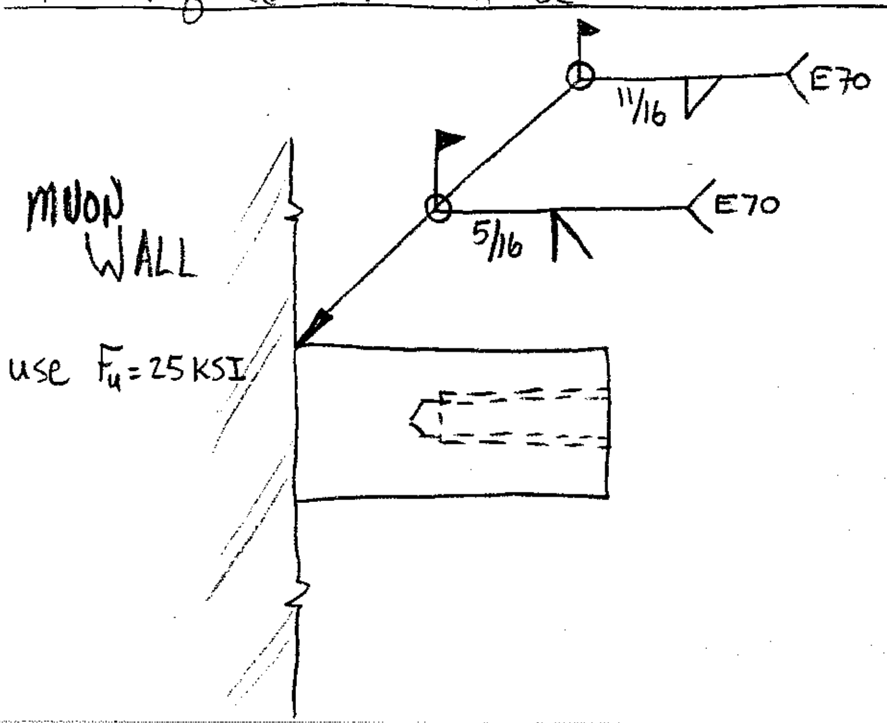
$$\Phi R_{nw} = .75a(.707)(25{,}000)(.60)$$

$$\Phi R_{nw} = 7{,}875\,a$$

The Required Weld size is:

$$\text{Required } a = \frac{R_u}{\Phi R_{nw}} = \frac{7{,}566.27\ \#/in}{7{,}875\ \#/in^2} = .96079\ in$$

The Required weld will be a 1" weld achieved as follows:



From: Steel Structures : Design and Behavior
3rd edition

where C_1 = coefficient for electrode = (Electrode used)/70
D = number of $\frac{1}{16}$s of an inch in weld size
L = length of vertical weld (in.)

(e) Summary. Compare the values of the service load P.

1. Strength analysis reducing P_n after completing analysis: $P = 57$ kips
3. ASD Manual tables: $P = 58$ kips

Note that ASD Manual tables used slightly different equations for the strength analysis than used in the LRFD Manual; however, the difference is not significant. The safe service load using ASD is 58 kips compared with 63 kips using LRFD. The LRFD value depends on the ratio of live to dead load whereas the ASD value is the same for all live to dead load ratios. The results from both LRFD and ASD agree favorably. ■

5.18 ECCENTRIC SHEAR CONNECTIONS —ELASTIC (VECTOR) ANALYSIS

The traditional elastic vector analysis is easier than the strength method to carry out when the computer is not available, or when the AISC Manual tables are not available. The elastic vector method is conservative, sometimes excessively so.

The elastic method has the following assumptions:

1. Each segment of weld, if of the same size, resists a concentrically applied load with an equal force. This concept was used for welds on tension members in Sec. 5.16.
2. The rotation caused by torsional moment is assumed to occur about the centroid of the weld configuration.
3. The load on a weld segment caused by the torsional moment is assumed to be proportional to the distance from the centroid of the weld configuration.
4. The direction of the force on a weld segment caused by torsion is assumed to be perpendicular to the radial distance from the centroid of the weld configuration.
5. The components of the forces due to direct load and due to torsion are combined vectorially to obtain a resultant force.

It will be convenient to think of this analysis using the principles of mechanics on a homogeneous material, combining direct shear with torsion. Beginning with the stresses on a homogeneous section,

$$f' = \frac{P}{A} = \text{stress due to direct shear} \qquad (5.18.1)$$

$$f'' = \frac{Tr}{I_p} = \text{stress due to torsional moment} \qquad (5.18.2)$$

where r = radial distance from the centroid to point of stress
I_p = polar moment of inertia

For computing nominal stresses or forces on weld segments the *locations* of the lines of weld are defined by edges along which the fillets are placed, rather than to the center of the effective throat. This makes little difference, since the throat dimension is usually small.

For the general case shown in Fig. 5.18.1, the components of stress due to direct shear are

$$f_x' = \frac{P_x}{A} \tag{5.18.3a}$$

$$f_y' = \frac{P_y}{A} \tag{5.18.3b}$$

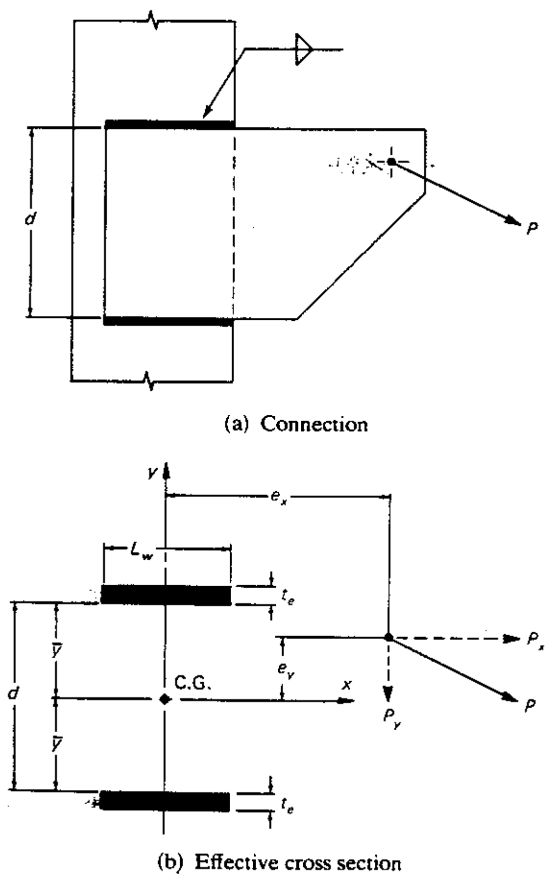


Figure 5.18.1 Eccentric bracket connection.

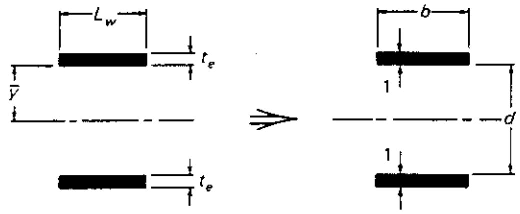


Figure 5.18.2 Treatment of weld configuration as lines having unit thickness.

The x- and y-components of f'' resulting from torsion are

$$f_x'' = \frac{Ty}{I_p} = \frac{(P_x e_y + P_y e_x)y}{I_p} \tag{5.18.4a}$$

$$f_y'' = \frac{Tx}{I_p} = \frac{(P_x e_y + P_y e_x)x}{I_p} \tag{5.18.4b}$$

where $$I_p = I_x + I_y = \sum I_{xx} + \sum A\bar{y}^2 + \sum I_{yy} + \sum A\bar{x}^2 \tag{5.18.5}$$

In Eq. 5.18.5, $\bar{x}$ and $\bar{y}$ refer to distances from the center of gravity of the weld group to the center of gravity of the individual weld segments. I_{xx} and I_{yy} refer to the moments of inertia of the individual segments with respect to their own centroidal axes.

Thus, for the situation of Fig. 5.18.2, Eq. 5.18.5 becomes

$$I_p = 2\left[\frac{L_w(t_e)^3}{12}\right] + 2\left[L_w(t_e)(\bar{y})^2\right] + 2\left[\frac{t_e(L_w)^3}{12}\right]$$

$$= \frac{t_e}{6}\left[L_w(t_e)^2 + 12L_w(\bar{y})^2 + L_w^3\right] \tag{5.18.6}$$

For practical situations, the first term of Eq. 5.18.6 is neglected because, with t_e small, the term is not significant compared to the other terms. Hence

$$I_p \approx \frac{t_e}{6}\left[12L_w(\bar{y})^2 + L_w^3\right] \tag{5.18.7}$$

Note that I_p equals the throat thickness t_e times the property of *lines*; i.e., an element having length but having a width of unity. Actually, the area A in Eqs. 5.18.3 equals the thickness t_e times the total length of the weld configuration; and in Eq. 5.18.5 the polar moment of inertia equals the thickness t_e times the polar moment of inertia of the configuration as *lines*. When the stress f is multiplied by t_e, it becomes a force R per unit length, say, kips/in.

Treating the welds making up the effective cross-section in Fig. 5.18.2 as line welds (i.e., as in deriving Eq. 5.18.7 with $t_e = 1$) and using the general terms b and d, as shown in Fig. 5.18.2, Eq. 5.18.7 becomes

$$I_p \approx \frac{1}{6}\left[12b\left(\frac{d}{2}\right)^2 + b^3\right] = \frac{b}{6}[3d^2 + b^2] \tag{5.18.8}$$

Table 5.18.1 gives I_p values treated as properties of lines for other common weld configurations.

■ EXAMPLE 5.18.1

Compute the maximum load (kips/in.) on the weld configuration shown for the bracket in Fig. 5.18.3 using the elastic (vector) method. Assume the plate thickness does not affect the result.

SOLUTION

The maximum force R will occur at points A and B. The properties of lines will be used.

(a) Locate the centroid of the configuration. Taking moments about the vertical weld,

$$\bar{x} = \frac{2(6)3}{2(6) + 8} = 1.8 \text{ in.}$$

(b) Compute the area (length) and the polar moment of inertia about the centroid of the configuration.

$L = 2(6) + 8 = 20$ in.

$$I_p = \underbrace{\frac{(8)^3}{12} + 2\left[6(4)^2\right]}_{I_x} + \underbrace{2\left[\frac{(6)^3}{12}\right] + 2\left[6(1.2)^2\right] + 8(1.8)^2}_{I_y} = 314 \text{ in.}^3$$

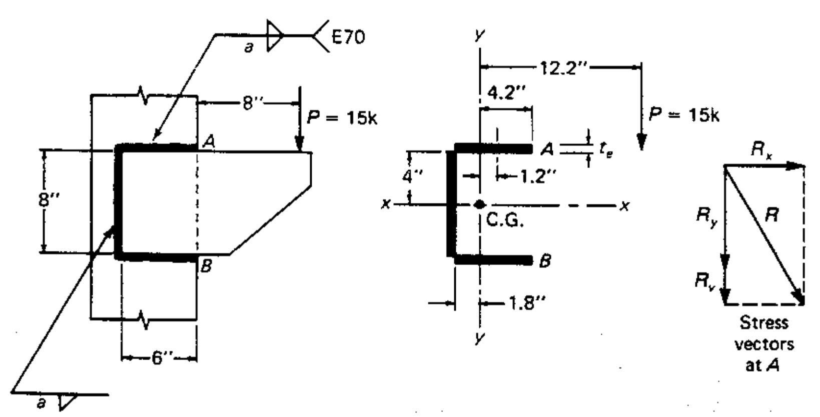
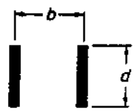
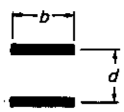
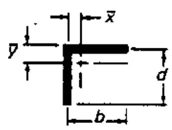
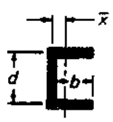
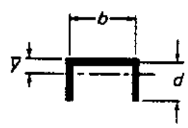
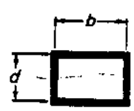
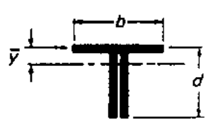
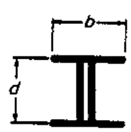
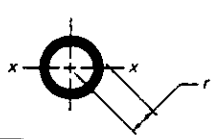


Figure 5.18.3 Example 5.18.1.

TABLE 5.18.1 PROPERTIES OF WELDS TREATED AS LINES

Section b = width; d = depth		Section Modulus $I_x/\bar{y}$	Polar Moment of Inertia, I_p about Center of Gravity
1. 		$S = \dfrac{d^2}{6}$	$I_p = \dfrac{d^3}{12}$
2. 		$S = \dfrac{d^2}{3}$	$I_p = \dfrac{d(3b^2+d^2)}{6}$
3. 		$S = bd$	$I_p = \dfrac{b(3d^2+b^2)}{6}$
4. 	$\bar{y} = \dfrac{d^2}{2(b+d)}$ $\bar{x} = \dfrac{b^2}{2(b+d)}$	$S = \dfrac{4bd+d^2}{6}$	$I_p = \dfrac{(b+d)^4-6b^2d^2}{12(b+d)}$
5. 	$\bar{x} = \dfrac{b^2}{2b+d}$	$S = bd + \dfrac{d^2}{6}$	$I_p = \dfrac{8b^3+6bd^2+d^3}{12} - \dfrac{b^4}{2b+d}$
6. 	$\bar{y} = \dfrac{d^2}{b+2d}$	$S = \dfrac{2bd+d^2}{3}$	$I_p = \dfrac{b^3+6b^2d+8d^3}{12} - \dfrac{d^4}{2d+b}$
7. 		$S = bd + \dfrac{d^2}{3}$	$I_p = \dfrac{(b+d)^3}{6}$
8. 	$\bar{y} = \dfrac{d^2}{b+2d}$	$S = \dfrac{2bd+d^2}{3}$	$I_p = \dfrac{b^3+8d^3}{12} - \dfrac{d^4}{b+2d}$
9. 		$S = bd + \dfrac{d^2}{3}$	$I_p = \dfrac{b^3+3bd^2+d^3}{6}$
10. 		$S = \pi r^2$	$I_p = 2\pi r^3$

(c) Compute the components of the force on the weld at points A and B. From the direct shear,

$$R_v = \frac{P}{L} = \frac{15}{20} = 0.75 \text{ kips/in.} \downarrow$$

From the torsion T about the centroid of the configuration,

$$R_x = \frac{Ty}{I_p} = \frac{15(12.2)4}{314} = 2.33 \text{ kips/in.} \rightarrow$$

$$R_y = \frac{Tx}{I_p} = \frac{15(12.2)4.2}{314} = 2.45 \text{ kips/in.} \downarrow$$

The vector sum gives the resultant force R,

$$R = \sqrt{(2.33)^2 + (2.45 + 0.75)^2} = 3.96 \text{ kips/in.}$$ ■

■ EXAMPLE 5.18.2

Determine the weld size required for the bracket of Fig. 5.18.3 when the service load P is 15 kips (80% live load and 20% dead load). Compare the results using (a) elastic (vector) analysis from Example 5.18.1 and (b) strength analysis as described in Sec. 5.17, both with AISC Load and Resistance Factor Design. Assume the plate thickness does not affect the result.

SOLUTION

(a) Elastic (vector) method. According to LRFD the factored load must be computed,

$$P_u = 1.2D + 1.6L = 1.2(0.2)15 + 1.6(0.8)15 = 22.8 \text{ kips}$$

The maximum load (kips/in.) on the weld due to the factored P will be (using the result from Example 5.18.1 for $P = 15$ kips),

$$R_u = 3.96(22.8/15) = 6.02 \text{ kips/in.}$$

$$\text{Weld resistance } \phi R_{nw} = 0.75a(0.707)(0.60F_{EXX})$$
$$= 0.75a(0.707)(0.60)70 = 22.3a$$

The weld size required is then

$$\text{Required } a = \frac{R_u}{\phi R_{nw}} = \frac{6.02}{22.3} = 0.27 \text{ in., say } \frac{5}{16} \text{ in.}$$

Use $\frac{5}{16}$-in. E70 fillet welds.

(b) Strength analysis method. First, the strength analysis must be performed. Divide the horizontal 6-in. weld into 6 parts and the vertical 8-in. length into 16 segments of $\frac{1}{2}$ in. each, though it is considered adequate to always use 1-in. segments. The modification of the strength method is used wherein R_i is taken equal to 7.42 kips/in. whenever the computed R_i would exceed that value. The instantaneous center is found by trial to be at -0.115 in. from the vertical weld line; that is, negative means toward the location of the applied load P. The solution details are in Tables 5.18.2 and 5.18.3.

TABLE 5.18.2 GEOMETRY AND $R_{i,\text{ULT}}$. EXAMPLE 5.18.2

Seg No.	Length (in.)	x (in.)	y (in.)	r_i (in.)	θ_1 (deg)	$\Delta_{i\,\max}$ (in.)	Δ_i (in.)	$R_{i,\text{ult}}$ (kips)
1	1	0.39	4.00	4.02	5.5	0.0776	0.0208	11.491
2	1	1.39	4.00	4.23	19.1	0.0525	0.0219	13.487
3	1	2.39	4.00	4.66	30.8	0.0436	0.0241	14.299
4	1	3.39	4.00	5.24	40.2	0.0391	0.0271	14.712
5	1	4.39	4.00	5.94	47.6	0.0364	0.0307	14.952
6	1	5.39	4.00	6.71	53.4	0.0347	0.0347	15.104
7	0.50	−0.12	3.75	3.75	−88.2	0.0278	0.0194	7.835
8	0.50	−0.12	3.25	3.25	−88.0	0.0278	0.0168	7.834
9	0.50	−0.12	2.75	2.75	−87.6	0.0279	0.0142	7.832
10	0.50	−0.12	2.25	2.25	−87.1	0.0280	0.0116	7.829
11	0.50	−0.12	1.75	1.75	−86.2	0.0281	0.0091	7.824
12	0.50	−0.12	1.25	1.26	−84.7	0.0283	0.0065	7.815
13	0.50	−0.12	0.75	0.76	−81.3	0.0288	0.0039	7.795
14	0.50	−0.12	0.25	0.28	−65.3	0.0318	0.0014	7.676

TABLE 5.18.3 SOLUTION FOR EXAMPLE 5.18.2 (b) ($r_0 = -0.115$ in.)

Seg No.	Computed R_i (kips)	Used R_i (kips)	$(R_i)_y$ (kips)	$R_i r_i$ (kips-in.)
1	10.49	7.42	0.71	29.83
2	12.53	7.42	2.32	31.27
3	13.61	7.42	3.72	34.32
4	14.29	7.42	4.74	38.59
5	14.73	7.42	5.45	43.70
6	14.99	7.42	5.93	49.41
7	7.62	3.72	−0.11	13.93
8	7.47	3.72	−0.13	12.07
9	7.22	3.72	−0.16	10.22
10	6.80	3.72	−0.19	8.36
11	6.10	3.72	−0.24	6.51
12	4.99	3.72	−0.34	4.66
13	3.35	3.35	−0.50	2.54
14	1.46	1.46	−0.61	0.40
			$\Sigma = 20.59$	285.80

From Eq. 5.17.9, and multiplying by 2 because of symmetry, gives

$$P_n = \frac{\Sigma R_i r_i}{e + r_0} = \frac{2(285.80)}{14.0 - 0.115} = 41.2 \text{ kips}$$

$$P_n = \Sigma (R_i)_y = 2(20.59) = 41.2 \text{ kips}$$

The nominal strength $P_n = 41.2$ kips. This is the strength using $\frac{1}{4}$-in. weld using E70 electrodes with the SMAW process. The design strength ϕP_n is $0.75(41.2) = 30.9$ kips. The weld size required is

$$P_u = 22.8 \text{ kips} = \phi P_n = 30.9 \frac{a}{0.25}$$

$$\text{Required weld size } a = \frac{22.8(0.25)}{30.9} = 0.18 \text{ in.}$$

(c) Use *LRFD Manual* [1.17] tables, p. 5-103. For $\frac{1}{4}$-in. weld using E70 electrodes,

$$a = (e - xL)/L = (14.0 - 1.80)/8 = 1.525$$
$$k = kL/L \quad = 6.0/8 = 0.75$$

	$k = 0.7$	0.75	0.8	
$a = 1.4$	0.983		1.11	
1.525	0.893	0.952	1.011	$C = 0.952$
1.6	0.874		0.99	

$$\text{Table value} = \phi P_n = CC_1DL = 0.952(1.0)(4)8 = 30.5 \text{ kips}$$

As may be noted, the LRFD tables give essentially the same result as obtained in part (b).

(d) Summary. The weld size a required using E70 electrodes and the SMAW process is

Elastic (vector) method, required $a = \frac{5}{16}$ in.

Strength analysis, required $a = \frac{3}{16}$ in.

As will always be the case, the elastic vector method is conservative. ■

5.19 LOADS APPLIED ECCENTRIC TO THE PLANE OF WELDS

When an applied load is eccentric to the *plane* of the weld configuration, as in Fig. 5.19.1, the strength method of analysis may still be used as long as the plane of the welds is rigid. The weld plane is rigid in Fig. 5.19.1 because the welds are on each side of a plate; i.e., there is sufficient rigidity between the two lines of weld such that there will be no bending *of the material being welded* in the plane of the welds.

As discussed in Sec. 5.17, the strength of a segment of weld depends on the angle θ_i of the resisting force R_i to the axis of the weld. It makes no difference whether R_i acts at an angle to the plane of the welds (Fig. 5.19.1) or whether it acts in the plane of the welds (eccentric shear as in Secs. 5.17 and 5.18).