Fermilab

SDC SOLENOID DESIGN NOTE #156

TITLE: Buckling Analysis of a Ring-Stiffened SDC Vacuum Shell

AUTHOR: Lyle W. Swenson, Jr., PS Associates, Inc.

DATE: August 30, 1991

Buckling Analysis of a Ring-Stiffened SDC Vacumm Shell (Task 5)

Prepared For

Fermilab (PO B14690)

August 30, 1991

Prepared by

Lyle W. Swenson, Jr.

PS Associates, Inc.
5755 Oberlin Drive, Suite 300
San Diego, CA 92121-4717

1.0 Introduction

This report documents buckling analyses performed to evaluate the feasibility of designing the SDC vacuum shell using Isogrid geometry with a overall thickness of 1.10 inches and stiffening rings spaced 45 inches apart. The following presentation is contained in four sections and two appendices.

Section 2.0 is a description of the material properties, model geometry, and loading used in the buckling analyses presented in this report.

Section 3.0 presents calculations for the stiffening ring minimum area moment of inertia following procedures outlined in the Compressed Gas Association Standard CGA-341 [2]. A derivation for the ring minimum moment of inertia is presented and compared with the CGA formulae.

Sections 4.0 presents results from a parametric study of stiffening ring geometry required to produce a critical buckling pressure of 30 psi. The study uses equations suggested in NASA SP-8007 [3] for predicting external pressure buckling of general orthotroptic cylindrical shells. The equations can be tailored for ring stiffened shells; these are the forms used in this report.

Appendix A is a copy of the relevant portions of NASA SP-8007 provided so the reader can examine in detail the equations used in the study presented in Section 4.0.

Finally, Appendix B contains a C language listing of the code written by the author to perform the numerical computations required for the buckling analysis presented in Section 4.0. A sample dataset is also included.

2.0 Modelling of the SDC Vacuum Shell

2.1 Material Properties

The vacuum shell structure is considered made from aluminum 2219-T851. Properties for this material are taken from MIL-HDBK-5C [1] and are given in Table 2-1 below.

Table 2-1. Properties for Al 2219-T851 (Room Temperature)

Young' Modulus	10.5×10^6 psi
Poisson's Ratio	0.33
Weight Density	0.102 lb/in^3
Tensile Ultimate Strength	62,000 psi
Tensile Yield Stress	47,000 psi
Allowable Stress	15,500 psi

For buckling analyses only the value of Young' modulus is required.

The stiffening ring is assumed to be made from carbon-fiber reinforced plastic (CFRP) with a Young's modulus value of 30×10^6.

2.2 Model Geometry

Basic geometry for the vacuum shell was specified by Fermilab as presented in Table 2-2 below.

Table 2-2. Basic Vacuum Shell Geometry

Shell Diameter	160 in
Shell Length	360 in
Ring Spacing	45 in

The Isogrid design geometry is given below. See Appendix A in the Task 1 Report for a discussion of the dimensionless parameters and computation of section details.

Table 2-3. Isogrid Geometry

Node Spacing	a	6.9000 in
Web Width	b	0.0950 in
Flange Depth	c	0.1100 in
Web Depth	d	0.8300 in
Triangle Height	h	5.9756 in
Flange Width	w	0.4500 in
Skin Thickness	t	0.1600 in

Table 2-4. Isogrid Dimensionless Parameters

$\alpha = bd/ht$	0.0825
β	6.0161
$\delta = d/t$	5.1875
$\tau = c/t$	0.6875
$\mu = wc/ht$	0.0518

Table 2-5. Isogrid Section Parameters

Effective Area	A	0.1815 in^2/in
Flange Fiber Distance	Cf	0.9400 in
Skin Fiber Distance	Cs	0.1600 in
Shell Modulus	E*	2.25×10^6 psi
Bending Rigidity	D	1.29×10^5 in-lb
Moment of Inertia	I	0.0109 in^4/in
Extension Stiffness	K	2.14×10^6 lb/in
Weight Thickness	t_{bar}	0.2244 in
Effective Thickness	t_{eff}	0.1815 in
Shell Thickness	t*	0.8486 in

2.3 Applied Loading

The only loading considered in the buckling analysis is an external lateral pressure of 30 psi.

3.0 Minimum Stiffening Ring Moment of Inertia via CGA Standard

CGA-341 [2] requires that each stiffening ring have a minimum area moment of inertia determined by either of the formulae:

$$I = 1.05\ D^3\ B/E_R \qquad (1)$$

or

$$I' = 1.38\ D^3\ B/E_R \qquad (2)$$

where,

I is the required moment of inertia of the ring about a centroidal axis parallel to the shell axis;

I' is the required moment of inertia of the combined section of ring and effective width of shell about a centroidal axis;

D is the outside diameter of the shell;

B is the stiffening ring spacing, and;

E_R is Young's modulus of the stiffening ring.

Substituting values from Section 2.0 into equations (1) and (2) yields:

$$I = 1.05\ (160)^3\ (45)\ /\ (30 \times 10^6)$$

$$I = 6.4512\ in^4$$

and

$$I' = 1.38\ (160)^3\ (45)\ /\ (30 \times 10^6)$$

$$I' = 8.4787\ in^4.$$

To be honest, I am not clear on the source of these equations. To satisfy my own curiosity, I developed an equation for the minimum ring moment of inertia based on ring stability.

The classical stability equation for a circular ring under external pressure was derived by Lévy [4] as:

$$p' = 3\ E_R\ I/(R^3\ b), \qquad (3)$$

were p' is the external pressure acting on the ring, E_R and I have the same meanings as above, R is the mean ring radius, and b is the effective width of the ring. Using the expression, R = D/2, equation (4) becomes:

$$p' = 24\ E_R\ I/(D^3\ b). \qquad (4)$$

In the worst case, when the shell buckles all the pressure loading is carried by the rings. In this case, pressure loading on each ring is given by:

$$p' = p\ B/b, \qquad (5)$$

where p is the external pressure.

When equation (5) is substituted into (4), and the result solved for the ring centroidal inertia, one obtains:

$$I = p\ D^3\ B/(24\ E_R), \qquad (6)$$

or, using a value p = 30 psi,

$$I = 1.25\ D^3\ B/E_R. \qquad (7)$$

This result can be compared to equation (1). As can be seen, the coefficient in equation (7) is 19 percent larger that the coefficient in equation (1).

4.0 Ring Geometry for a Given Critical Buckling Pressure

NASA SP-8003 [3] provides equations for computing the critical buckling pressure for orthotropic cylindrical shells. Ring stiffened shells can be analyzed as a special case of these equations. Appendix A is a copy of the NASA SP-8007 material discussing the buckling equations used in this report; development of these buckling equations follows from the work of Jones [5]. Appendix B gives a listing of the computer code developed to evaluate the buckling equations.

The model used in the analysis considered the Isogrid geometry given in Section 2.2 and shown in Figure 5.1. The stiffener ring is assumed to have the geometry shown in Figure 5.2. Results given in the table below use a ring spacing of 45 inches and a critical buckling pressure of 30 psi. Buckling pressure computed using the equations in NASA SP-8007 includes a correlation coefficient reduction factor of 0.75.

Shown in Table 5-1 are values for ring geometry as functions of ring height, cf. Figure 5.2. The values in the table are: ring thickness (t_R), ring width (w_R), ring cross-sectional area (A_R), and ring moment of inertia (I_R) about the shell reference surface.

These data were computed in the following manner. For each ring height, the maximum allowable ring thickness was used to determine the minimum ring width based on a ring-shell buckling pressure of 30 psi.

The ring geometry presented in Table 5-1 are plotted in Figures 5.3 through 5.6 as functions of ring height. It was desired that the ring height be kept to 1.1 inches. However, even a ring height of 1.5 inches requires a ring width of 32 inches! Consequently, greater ring eccentricity is needed to achieved the ring moment of inertia required to provide stability. As seen from the data, or graphically in

Table 5-1 Ring Geometry vs. Ring Height

Height	Thickness	Width	Area	Inertia
1.50	0.40	32.00	12.80	16.81
1.75	0.65	8.10	5.27	8.61
2.00	0.90	3.48	3.13	6.26
2.25	1.15	1.87	2.15	5.17
2.50	1.40	1.15	1.61	4.59
2.75	1.65	0.76	1.25	4.19
3.00	1.90	0.53	1.00	3.90

Figure 5.4, the minimum required ring width decreases rapidly with ring height.

References

[1] MIL-HDBK-5C, Military Standardization Handbook, Metallic Materials and Elements for Aerospace Vehicle Structures. September 15, 1976.

[2] CGA-341, "Standard for Insulated Cargo Tank Specification for Cryogenic Liquids", Compressed Gas Association. Third Edition, 1987.

[3] NASA SP-8007, "Buckling of Thin-walled Circular Cylinders", National Aeronautics and Space Administration, September 1965, Revised August 1968.

[4] Lévy, M., J. Math. Pure Appl. (Liouville), Ser. 3, Vol. 10, p.5, 1884.

[5] Jones, R.M., "Buckling of Circular Cylindrical Shells with Multiple Orthotropic Layers and Eccentric Stiffeners", AIAA J., Vol. 6(2), pp. 2301-2305, Dec. 1968.

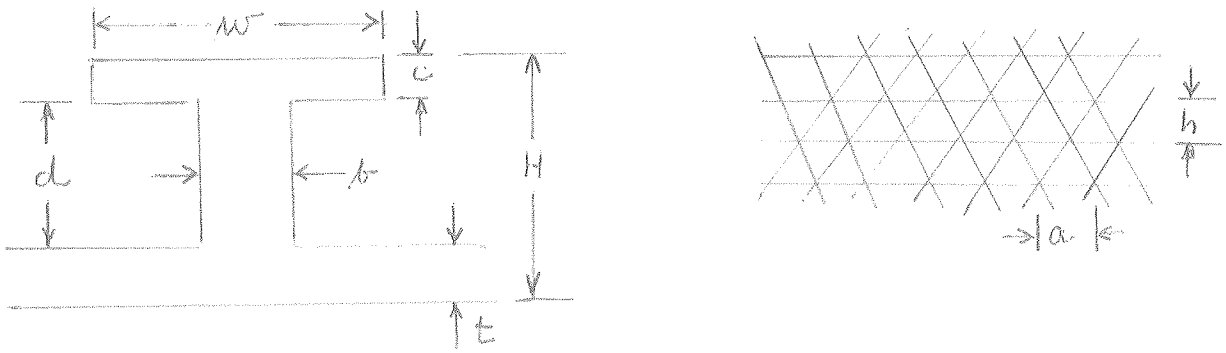

Figure 5.1 Isogrid Geometry

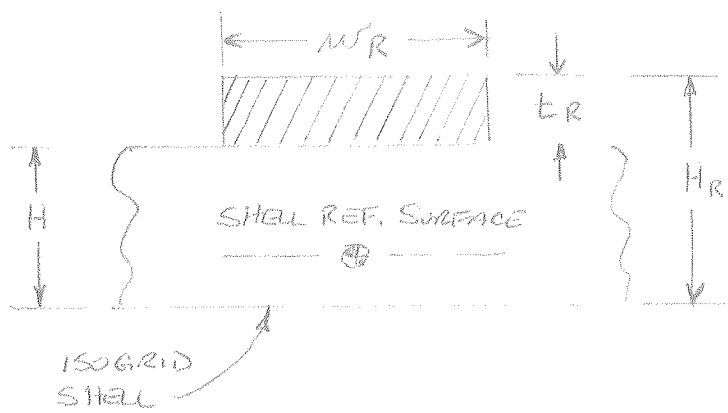

Figure 5.2 Shell-Ring Section Geometry

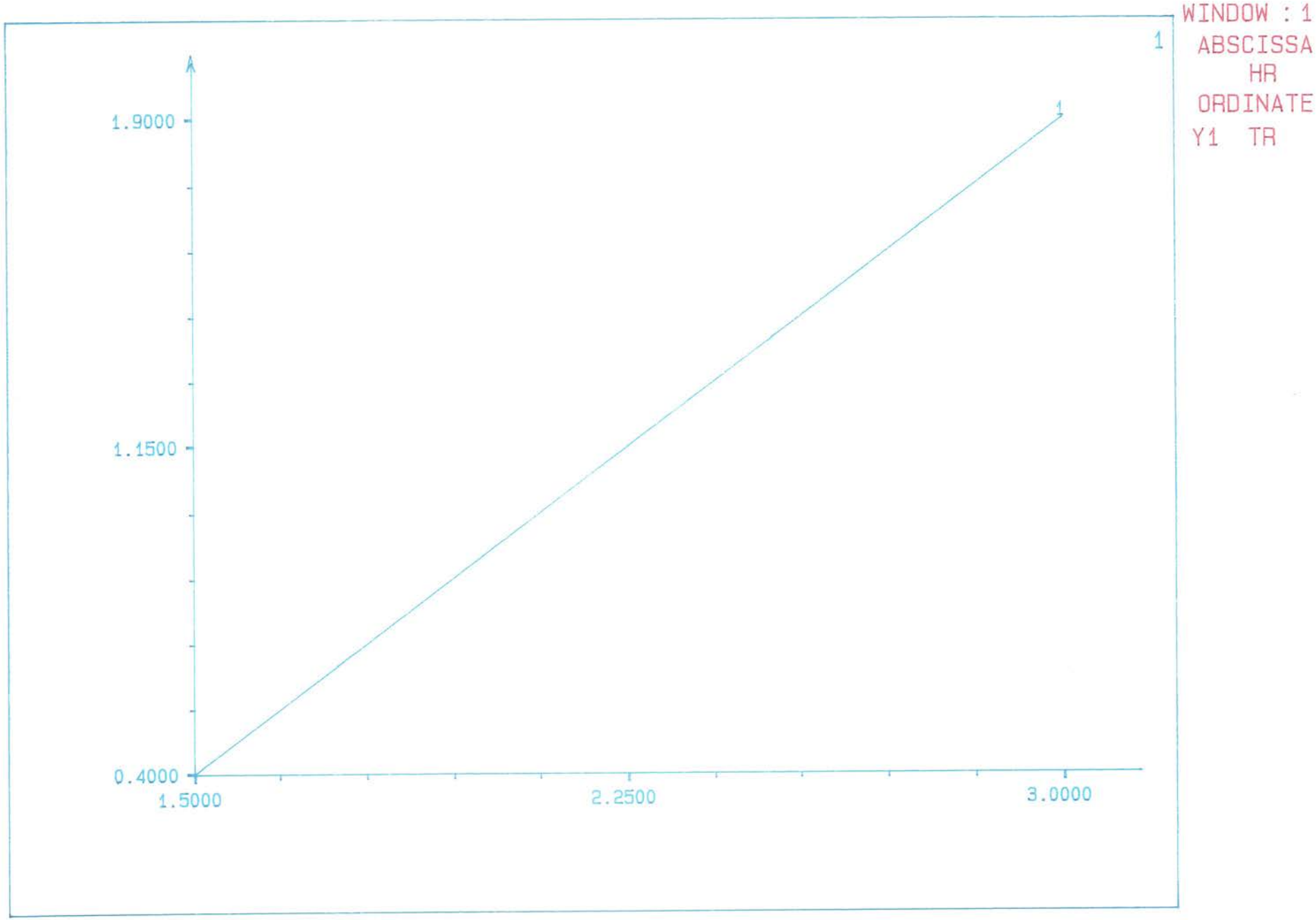

Figure 5.3 Ring Thickness vs. Ring Height

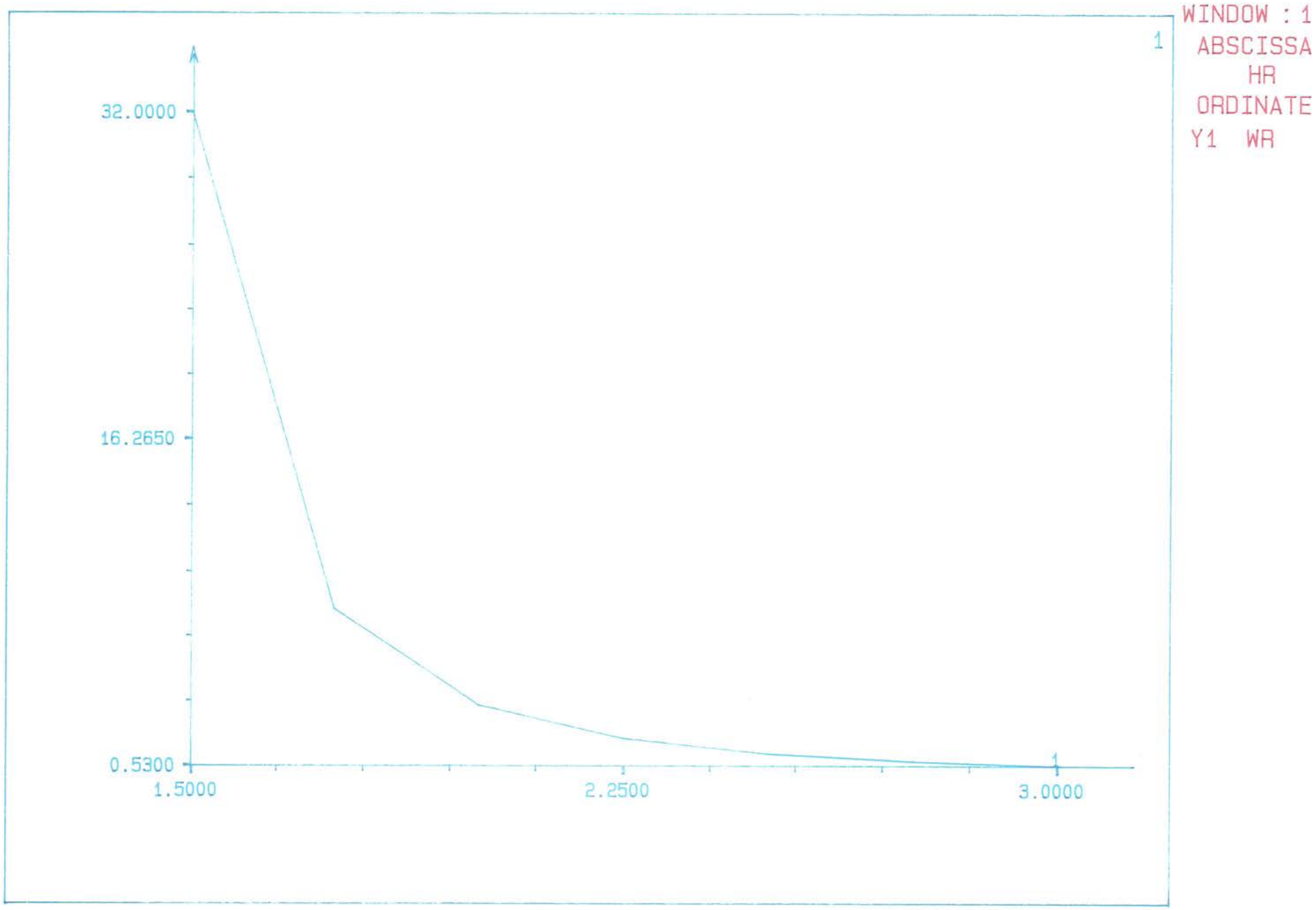

Figure 5.4 Ring Width vs. Ring Height

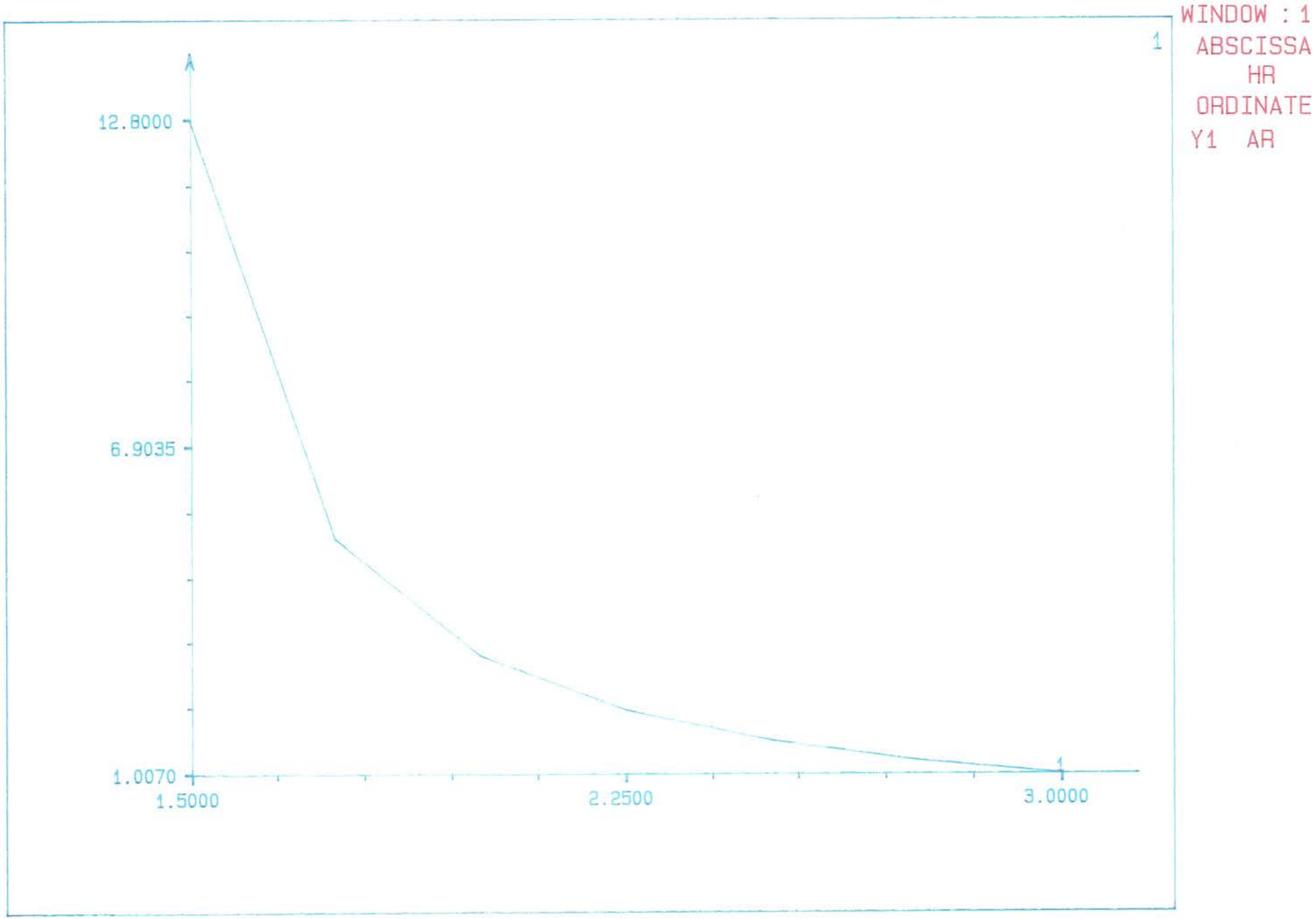

Figure 5.5 Ring Area vs. Ring Height

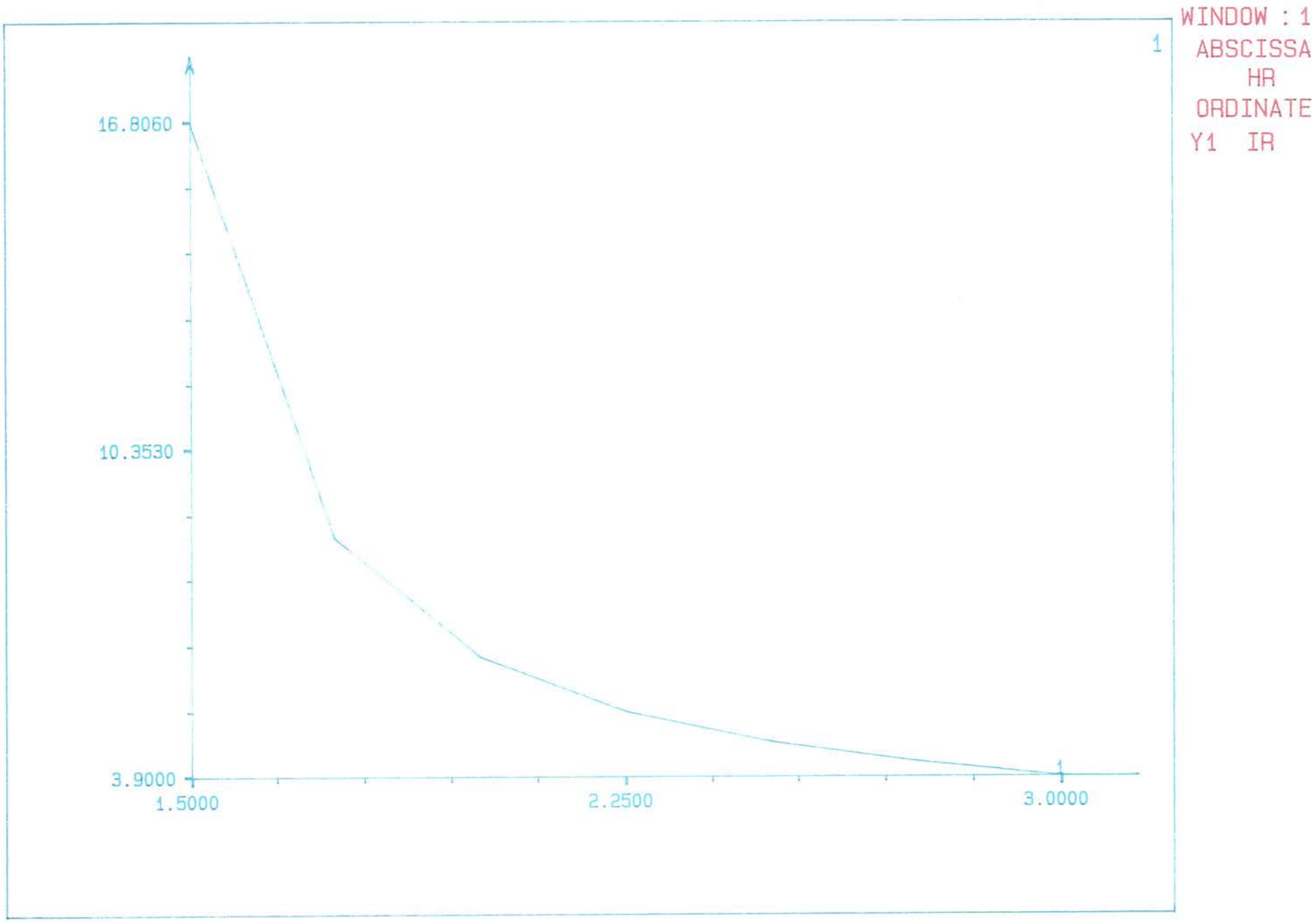

Figure 5.6 Ring Inertia (about shell reference) vs. Ring Height

Appendix A

This appendix contains selected pages from NASA SP-8007, "Buckling of thin-walled circular cylinders", that pertain to buckling analysis of orthotropic shells. The equations discussed in this document were tailored for ring stiffened shell analysis as presented in Section 4.0.

NASA
SPACE VEHICLE
DESIGN CRITERIA
(STRUCTURES)

NASA SP-8007

BUCKLING OF THIN-WALLED CIRCULAR CYLINDERS

SEPTEMBER 1965
Revised
AUGUST 1968

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

CONTENTS

SYMBOLS vii

1. INTRODUCTION 1

2. STATE OF THE ART 2

3. CRITERIA 3
   3.1 General 3
   3.2 Guides for Compliance 3

4. RECOMMENDED PRACTICES 3
   4.1 Scope 3
   4.2 Isotropic Unstiffened Cylinders 4
      4.2.1 Axial Compression 4
      4.2.2 Bending 7
      4.2.3 External Pressure 8
      4.2.4 Torsion 11
      4.2.5 Combined Loads 13
         4.2.5.1 Circular Cylinders in Axial Compression and Bending 13
         4.2.5.2 Circular Cylinders in Axial Compression and External Pressure 13
         4.2.5.3 Circular Cylinders in Axial Compression and Torsion 14
         4.2.5.4 Internally Pressurized Circular Cylinders in Axial Compression 14
         4.2.5.5 Internally Pressurized Circular Cylinders in Bending 15
         4.2.5.6 Internally Pressurized Cylinders in Axial Compression & Bending 16
   4.3 Orthotropic Cylinders 16
      4.3.1 Axial Compression 16
      4.3.2 Bending 19
      4.3.3 External Pressure 19
      4.3.4 Torsion 21

4.3.5 Combined Bending and Axial Compression 21
4.3.6 Elastic Constants 22
4.3.6.1 Stiffened Multilayered Orthotropic Cylinders 22
4.3.6.2 Isotropic Cylinders with Stiffeners and Rings 25
4.3.6.3 Ring-Stiffened Corrugated Cylinders 26
4.3.6.4 Waffle-Stiffened Cylinders . . . 26
4.3.6.5 Special Considerations 26

4.4 Isotropic Sandwich Cylinders 27
4.4.1 Axial Compression 28
4.4.2 Bending 32
4.4.3 Lateral Pressure 32
4.4.4 Torsion 33
4.5 Cylinders with an Elastic Core 35
4.5.1 Axial Compression 35
4.5.2 External Pressure 36
4.5.3 Torsion 39
4.5.4 Combined Axial Compression and Lateral Pressure 40
4.6 Design of Rings 41

REFERENCES 43

NASA SPACE VEHICLE DESIGN CRITERIA MONOGRAPHS ISSUED TO DATE 49

SYMBOLS

A_s, A_r	stiffener and ring area, respectively
B_1	extensional stiffness of isotropic sandwich wall
b	stiffener spacing
b_e	effective width of skin between stiffeners
$\overline{C}_x, \overline{C}_y, \overline{C}_{xy}, \overline{K}_{xy}$	coupling constants for orthotropic cylinders
c	coefficient of fixity in Euler column formula
D	wall flexural stiffness per unit width, $\dfrac{Et^3}{12(1-\mu^2)}$
D_q	transverse shear-stiffness parameter for isotropic sandwich wall, $G_{xz}\dfrac{h^2}{h-\frac{1}{2}(t_1+t_2)}$
$\overline{D}_x, \overline{D}_y$	bending stiffness per unit width of wall in x- and y-direction, respectively, $\overline{D}_x = \overline{D}_y = D$ for isotropic cylinder
$\overline{D}_{xy}$	modified twisting stiffness of wall; $\overline{D}_{xy} = 2D$ for isotropic cylinder
D_1	flexural stiffness of isotropic sandwich wall, $\dfrac{E_s t h^2}{2(1-\mu^2)}$
d	ring spacing
E	Young's modulus
E_c	Young's modulus of elastic core
E_R	reduced modulus
E_S	Young's modulus of face sheet of sandwich
E_r, E_s	Young's modulus of rings and stiffeners, respectively
E_{sec}	secant modulus for uniaxial stress-strain curve
E_{tan}	tangent modulus for uniaxial stress-strain curve
E_x, E_y	Young's modulus of orthotropic material in x- and y-direction, respectively

$\bar{E}_x, \bar{E}_y, \bar{E}_{xy}$	extensional stiffness of wall in x- and y-direction, respectively; $\bar{E}_x = \bar{E}_y = \frac{Et}{1-\mu^2}, \bar{E}_{xy} = \frac{\mu Et}{1-\mu^2}$ for isotropic cylinder
E_z	Young's modulus of sandwich core in direction perpendicular to face sheet of sandwich
G	shear modulus
G_s, G_r	shear modulus of stiffeners and rings, respectively
G_{xy}	inplane shear modulus of orthotropic material
$\bar{G}_{xy}$	shear stiffness of orthotropic or sandwich wall in x-y plane; $\bar{G}_{xy} = Gt$ for isotropic cylinder
G_{xz}, G_{yz}	shear modulus of core of sandwich wall in x-z and y-z planes, respectively
h	depth of sandwich wall measured between centroids of two face sheets
$\bar{I}$	moment of inertia per unit width of corrugated cylinder
I_r, I_s	moment of inertia of rings and stiffeners, respectively, about their centroid
J_r, J_s	beam torsion constant of rings and stiffeners, respectively
k_p	buckling coefficient of cylinder subject to hydrostatic pressure, $pr\,\ell^2/\pi^2 D$
k_{pc}	buckling coefficient of cylinder with an elastic core subject to lateral pressure, pr^3/D
k_x	buckling coefficient of cylinder subject to axial compression, $N_x\ell^2/\pi^2 D$ or $N_x\ell^2/\pi^2 D_1$
k_y	buckling coefficient of cylinder subject to lateral pressure, $N_y\ell^2/\pi^2 D$ or $N_y\ell^2/\pi^2 D_1$
k_{xy}	buckling coefficient of cylinder subjected to torsion, $N_{xy}\ell^2/\pi^2 D$ or $N_{xy}\ell^2/\pi^2 D_1$
ℓ	length of cylinder
M	bending moment on cylinder
M_{press}	bending moment at collapse of a pressurized cylinder
$M_{p=0}$	bending moment at collapse of a nonpressurized cylinder
M_t	twisting moment on cylinder
m	number of buckle half waves in the axial direction
N_o	$\frac{2\gamma E}{\sqrt{1-\mu^2}} \frac{h}{r} \sqrt{t_1 t_2}$
N_x	axial load per unit width of circumference for cylinder subjected to axial compression

N_y	circumferential load per unit width of circumference for cylinder subjected to lateral pressure
N_{xy}	shear load per unit width of circumference for cylinder subjected to torsion
n	number of buckle waves in the circumferential direction
P	axial load on cylinder
P_{press}	axial load on pressurized cylinder at buckling
$P_{p=0}$	axial load on nonpressurized cylinder at buckling
p	pressure, usually critical pressure
R	shear flexibility coefficient, $\frac{\pi^2 D}{\ell^2 D_q}$
R_b	ratio of bending moment on cylinder subjected to more than one type of loading to the allowable bending moment for the cylinder when subjected only to bending
R_c	ratio of axial load in cylinder subjected to more than one type of loading to the allowable axial load for the cylinder when subjected only to axial compression
R_p	ratio of external pressure on cylinder subjected to more than one type of loading to the allowable external pressure for the cylinder when subjected only to external pressure
R_t	ratio of torsional moment on cylinder subjected to more than one type of loading to the allowable torsional moment for the cylinder when subjected only to torsion
r	radius of cylinder
S	cell size of honeycomb core (see eq. 92)
t	skin thickness of isotropic cylinder; thickness of corrugated cylinder
$\bar{t}$	effective thickness of corrugated cylinder, area per unit width of circumference
t_f	face thickness of sandwich cylinder having equal thickness faces
t_k	skin thickness of kth layer of layered cylinder
t_1, t_2	facing-sheet thicknesses for sandwich construction having faces of unequal thickness
x,y,z	coordinates in the axial, circumferential, and radial directions, respectively
Z	curvature parameter; $\frac{\ell^2}{rt}\sqrt{1-\mu^2}$ for isotropic cylinder, $2\frac{\ell^2}{rh}\sqrt{1-\mu^2}$ for isotropic sandwich cylinder

$\tilde{z}_k$	distance of center of kth layer of layered cylinder from reference surface (positive outward)
$\tilde{z}_s, \tilde{z}_r$	distance of centroid of stiffeners and rings, respectively, from reference surface (positive when stiffeners or rings are on outside)
β	buckle aspect ratio $\left(\frac{n\ell}{\pi r m}\right)$
γ	correlation factor to account for difference between classical theory and predicted instability loads
Δ	distance of reference surface from inner surface of layered wall
$\Delta\gamma$	increase in buckling correlation factor due to internal pressure
δ	ratio of core density of honeycomb sandwich plate to density of face sheet of sandwich plate
δ_k	distance of centroid of kth layer of layered cylinder from inner surface
η	plasticity reduction factor
μ	Poisson's ratio
μ_c	Poisson's ratio of core material
μ_x, μ_y	Poisson's ratios associated with stretching of an orthotropic material in x- and y-directions, respectively
σ_p	critical axial stress for a cylinder with an elastic core
σ_x	axial stress
σ_y	circumferential stress
τ	shear stress
τ_{cr}	torsional buckling stress of an unfilled cylinder
τ_{xy}	shear stress in the x-y plane

4.3 Orthotropic Cylinders

The term "orthotropic cylinders" covers a wide variety of cylinders. In its strictest sense, it denotes cylinders made of a single orthotropic material or of orthotropic layers. It also denotes types of stiffened cylinders for which the stiffener spacing is small enough for the cylinder to be approximated by a fictitious sheet whose orthotropic bending and extensional properties include those of the individual stiffening elements averaged out over representative widths or areas. Generally, the directions of the axes of orthotropy are taken to coincide with the longitudinal and circumferential directions of the cylinder.

The behavior of the various types of orthotropic cylinders may be described by a single theory, the elements of which are equations of equilibrium for the buckled structure, relationships between force and moment resultants, and extensional and bending strains. For cylinders of a single orthotropic material, it is generally permissible to neglect coupling between force resultants and bending strains, and between moment resultants and extensional strains. The theory is then similar to that for isotropic cylinders. For stiffened cylinders or for cylinders having orthotropic layers, however, the neglect of the coupling terms can lead to serious errors. For example, the inclusion of coupling terms yields a significant difference in theoretical results for stiffened cylinder configurations having stiffeners on the inner surface or on the outer surface. The difference vanishes when coupling is omitted.

Theoretical and experimental results for stiffened shells are generally in better agreement than those for unstiffened shells. The possibility of local buckling of the cylinder between stiffening elements should be checked.

4.3.1 Axial Compression

A buckling equation for stiffened orthotropic cylinders in compression (ref. 20) is given by:

$$N_x = \left(\frac{\ell}{m\pi}\right)^2 \frac{\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}} \text{ for } (n \geq 4) \tag{37}$$

in which

$$A_{11} = \overline{E}_x \left(\frac{m\pi}{\ell}\right)^2 + \overline{G}_{xy} \left(\frac{n}{r}\right)^2 \tag{38}$$

$$A_{22} = \overline{E}_y \left(\frac{n}{r}\right)^2 + \overline{G}_{xy} \left(\frac{m\pi}{\ell}\right)^2 \tag{39}$$

$$A_{33} = \overline{D}_x \left(\frac{m\pi}{\ell}\right)^4 + \overline{D}_{xy} \left(\frac{m\pi}{\ell}\right)^2 \left(\frac{n}{r}\right)^2 + \overline{D}_y \left(\frac{n}{r}\right)^4 + \frac{\overline{E}_y}{r^2} + \frac{2\overline{C}_y}{r} \left(\frac{n}{r}\right)^2 + \frac{2\overline{C}_{xy}}{r} \left(\frac{m\pi}{\ell}\right)^2 \tag{40}$$

$$A_{12} = A_{21} = \left(\overline{E}_{xy} + \overline{G}_{xy}\right) \frac{m\pi}{\ell} \frac{n}{r} \tag{41}$$

$$A_{23} = A_{32} = \left(\overline{C}_{xy} + 2\overline{K}_{xy}\right)\left(\frac{m\pi}{\ell}\right)^2 \frac{n}{r} + \frac{\overline{E}_y}{r} \frac{n}{r} + \overline{C}_y \left(\frac{n}{r}\right)^3 \tag{42}$$

$$A_{31} = A_{13} = \frac{\overline{E}_{xy}}{r} \frac{m\pi}{\ell} + \overline{C}_x \left(\frac{m\pi}{\ell}\right)^3 + \left(\overline{C}_{xy} + 2\overline{K}_{xy}\right) \frac{m\pi}{\ell} \left(\frac{n}{r}\right)^2 \tag{43}$$

Values of stiffnesses to be used for various types of construction are given in Section 4.3.6. Prebuckling deformations are not taken into account in the derivation of the equation. The cylinder edges are assumed to be supported by rings that are rigid in their own plane but offer no resistance to rotation or bending out of their plane. The equation can be specialized for various types of cylinders which have been treated separately in the literature; for example, unstiffened or stiffened orthotropic cylinders with eccentricity effects neglected (refs. 21 and 22), and unstiffened or stiffened orthotropic cylinders with eccentricity effects taken into account (refs. 23 to 27). For ring-stiffened corrugated cylinders, a similar but not identical theory is given in references 28 and 29. For given cylinder and stiffener dimensions, the values of m and n to be used are those which minimize $\overline{N}_x$.

The unusually large number of parameters in equation (37) does not permit any definitive numerical results to be shown. For combinations of parameters representative of stiffened shells, calculations indicate that external stiffening, whether stringers or rings, or both, can be more effective than internal stiffening for axial compression. Generally, calculations neglecting stiffener eccentricity yield unconservative values of the buckling load of internally stiffened cylinders and conservative values of the buckling load of externally stiffened cylinders (ref. 23). An extensive investigation of the variation of the buckling load with various stiffener parameters is reported in reference 25. The limited experimental data (refs. 28 to 35) for stiffened shells are in reasonably good agreement with the theoretical results for the range of parameters investigated.

On the basis of available data, it is recommended that the buckling loads of cylinders with closely spaced, moderately large stiffeners calculated from equation (37) be multiplied by a factor of 0.75. Correlation coefficients covering the transition from unstiffened cylinders to stiffened cylinders with closely spaced stiffeners have not been fully investigated. While theory and experiment (ref. 34) indicate that restraint against edge rotation and longitudinal movement significantly increases the buckling load, not enough is known about the edge restraint of actual cylinders to warrant taking advantage of these effects unless substantiated by tests.

For layered or unstiffened orthotropic cylindrical shells, the available test data are quite meager (refs. 36 and 37). For configurations where the coupling coefficients $\overline{C}_x$, $\overline{C}_y$, $\overline{C}_{xy}$, and $\overline{K}_{xy}$ can be neglected, it is recommended that the buckling load be calculated from the equation

$$\frac{N_x \ell^2}{\pi^2 \overline{D}_x} = m^2 \left(1 + 2 \frac{\overline{D}_{xy}}{\overline{D}_x} \beta^2 + \frac{\overline{D}_y}{\overline{D}_x} \beta^4\right) + \frac{\gamma^2 \ell^4}{\pi^4 m^2 \overline{D}_x r^2} \frac{\overline{E}_x \overline{E}_y - \overline{E}_{xy}^2}{\overline{E}_x + \left(\frac{\overline{E}_x \overline{E}_y - \overline{E}_{xy}^2}{\overline{G}_{xy}} - 2\overline{E}_{xy}\right) \beta^2 + \overline{E}_y \beta^4} \tag{44}$$

The correlation factor γ is taken to be of the same form as the correlation factor for isotropic cylinders [eq. (5)] with the thickness replaced by the geometric mean of the radii of gyration for the axial and circumferential directions. Thus

$$\gamma = 1 - 0.901 \left(1 - e^{-\phi}\right) \tag{45}$$

where

$$\phi = \frac{1}{29.8}\left[\frac{r}{\sqrt[4]{\frac{\bar{D}_x\bar{D}_y}{\bar{E}_x\bar{E}_y}}}\right]^{\frac{1}{2}} \tag{46}$$

4.3.2 Bending

Theoretical and experimental results (refs. 24, 29, and 38 to 40) indicate that the critical maximum load per unit circumference of a stiffened cylinder in bending can exceed the critical unit load in axial compression. In the absence of an extensive investigation, it is recommended that the critical maximum load per unit circumference of a cylinder with closely spaced stiffeners be taken as equal to the critical load in axial compression, which is calculated from equation (37) multiplied by a factor of 0.75.

For layered or unstiffened orthotropic cylinders with negligible coupling coefficients, it is recommended that the maximum unit load be calculated by means of equation (44) with γ replaced by

$$\gamma = 1 - 0.731\,(1 - e^{-\phi}) \tag{47}$$

where

$$\phi = \frac{1}{29.8}\left[\frac{r}{\sqrt[4]{\frac{\bar{D}_x\bar{D}_y}{\bar{E}_x\bar{E}_y}}}\right]^{\frac{1}{2}} \tag{48}$$

4.3.3 External Pressure

The counterpart of equation (37) for stiffened orthotropic cylinders under lateral pressure is given by

$$p = \frac{r}{n^2}\,\frac{\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}} \tag{49}$$

For hydrostatic pressure, the quantity n^2 shown in equation (49) is replaced by

$$n^2 + \frac{1}{2}\left(\frac{m\pi r}{\ell}\right)^2$$

In the case of lateral pressure, m is equal to unity while n must be varied to yield a minimum value of the critical pressure, but is not less than 2. In the case of hydrostatic pressure, the value of m should be varied as well as n. For long cylinders, equation (49) is replaced by

$$p = \frac{3\left(\overline{D}_y - \frac{\overline{C}_y^2}{\overline{E}_y}\right)}{r^3} \tag{50}$$

If the coupling coefficients $\overline{C}_x$, $\overline{C}_y$, $\overline{C}_{xy}$, and $\overline{K}_{xy}$ can be neglected, the critical buckling pressure can be approximated by (ref. 22):

$$p \approx \frac{5 \cdot 513}{\ell_r^{\frac{3}{2}}}\left[\frac{\overline{D}_y^3\left(\overline{E}_x\overline{E}_y - \overline{E}_{xy}^2\right)}{\overline{E}_y}\right]^{\frac{1}{4}} \tag{51}$$

for

$$\left(\frac{\overline{D}_y}{\overline{D}_x}\right)^{\frac{3}{2}}\left(\frac{\overline{E}_x\overline{E}_y - \overline{E}_{xy}^2}{12\,\overline{E}_y\overline{D}_x}\right)^{\frac{1}{2}}\frac{\ell^2}{r} > 500 \tag{52}$$

Equation (49) has been investigated primarily for isotropic cylinders with ring stiffeners (refs. 41 to 43). For closely spaced ring stiffening, references 41 and 42 show that the effectiveness of inside or outside rings depends on the shell and ring geometries. Generally, for shells with values of Z less than 100, outside rings are more effective than inside rings, while for values of Z greater than 500, the reverse is true. As the ring geometry varies, the effectiveness of outside stiffening tends to increase as the stiffness of the rings relative to the shell increases. Somewhat lower buckling pressures are given by the extremely complex but more accurate theory of reference 44, but the differences are not so significant as to warrant its use.

The experimental results for ring-stiffened cylinders described in reference 45 are in reasonably good agreement with the theoretical results of equation (49). For cylinders of all types, it is recommended that the buckling pressure calculated from equation (49) be multiplied by a factor of 0.75 for use in design, as has been recommended for unstiffened isotropic cylinders of moderate length.

4.3.4 Torsion

The problem of torsional buckling of orthotropic cylinders has been treated in references 22 and 46, which do not take coupling between bending and extension into account, and in reference 47, which does. If coupling effects are negligible, the critical torque of moderately long cylinders can be estimated from the relationship (ref. 22):

$$M_t \approx 21.75\, \overline{D}_y^{\frac{5}{8}} \left(\frac{\overline{E}_x \overline{E}_y - \overline{E}_{xy}^2}{\overline{E}_y} \right)^{\frac{3}{8}} \frac{R^{\frac{5}{4}}}{\ell^{\frac{1}{2}}} \tag{53}$$

for

$$\left(\frac{\overline{D}_y}{\overline{D}_x} \right)^{\frac{5}{6}} \left(\frac{\overline{E}_x \overline{E}_y - \overline{E}_{xy}^2}{12\, \overline{E}_y \overline{D}_x} \right)^{\frac{1}{2}} \frac{\ell^2}{r} \gtrsim 500 \tag{54}$$

Reference 47, however, shows that coupling effects are quite important for cylinders stiffened by closely spaced rings. For long shells, internal rings are generally more effective than outside rings; for short shells, the reverse is true. In the absence of general formulas or graphs to cover the entire range of parameters that should be considered, the equations of reference 47 should be solved for each specific case considered.

The test data of reference 48 are in good agreement with theoretical predictions but are insufficient to provide an adequate test of the theory. It is therefore recommended that theoretical critical torques be multiplied by a factor of 0.67 for moderately long cylinders.

4.3.5 Combined Bending and Axial Compression

On the basis of theory (refs. 24, 29, and 38) and limited experimental data (refs. 28 and 29), a straight-line interaction curve is recommended for orthotropic cylinders subjected to combined bending and axial compression. The critical combinations of loading are thus given by

$$R_c + R_b = 1 \tag{55}$$

4.3.6 Elastic Constants

The values of the various elastic constants used in the theory of buckling of orthotropic cylinders are different for different types of construction.

4.3.6.1 Stiffened Multilayered Orthotropic Cylinders

Some widely used expressions for this type of cylinder are:

$$\bar{E}_x = \sum_{k=1}^{N} \left(\frac{E_x}{1 - \mu_x \mu_y}\right)_k t_k + \frac{E_s A_s}{b} \tag{56}$$

$$\bar{E}_y = \sum_{k=1}^{N} \left(\frac{E_y}{1 - \mu_x \mu_y}\right)_k t_k + \frac{E_r A_r}{d} \tag{57}$$

$$\bar{E}_{xy} = \sum_{k=1}^{N} \left(\frac{\mu_x E_y}{1 - \mu_x \mu_y}\right)_k t_k = \sum_{k=1}^{N} \left(\frac{\mu_y E_x}{1 - \mu_x \mu_y}\right)_k t_k \tag{58}$$

$$\bar{G}_{xy} = \sum_{k=1}^{N} \left(G_{xy}\right)_k t_k \tag{59}$$

$$\bar{D}_x = \sum_{k=1}^{N} \left(\frac{E_x}{1 - \mu_x \mu_y}\right)_k \left(\frac{1}{12} t_k^3 + t_k \tilde{z}_k^2\right) + \frac{E_s I_s}{b} + \tilde{z}_s^2 \frac{E_s A_s}{b} \tag{60}$$

$$\bar{D}_y = \sum_{k=1}^{N} \left(\frac{E_y}{1-\mu_x\mu_y}\right)_k \left(\frac{1}{12}\, t_k^3 + t_k \tilde{z}_k^2\right) + \frac{E_r I_r}{b} + \tilde{z}_r^2 \frac{E_r A_r}{d} \tag{61}$$

$$\bar{D}_{xy} = \sum_{k=1}^{N} \left(4G_{xy} + \frac{\mu_x E_y}{1-\mu_x\mu_y} + \frac{\mu_y E_x}{1-\mu_x\mu_y}\right)_k \frac{1}{12}\, t_k^3 + t_k \tilde{z}_k^2$$

$$+ \frac{G_s J_s}{b} + \frac{G_r J_r}{d} \tag{62}$$

$$\bar{C}_x = \sum_{k=1}^{N} \left(\frac{E_x}{1-\mu_x\mu_y}\right)_k t_k \tilde{z}_k + \tilde{z}_s \frac{E_s A_s}{b} \tag{63}$$

$$\bar{C}_y = \sum_{k=1}^{N} \left(\frac{E_y}{1-\mu_x\mu_y}\right)_k t_k \tilde{z}_k + \tilde{z}_r \frac{E_r A_r}{d} \tag{64}$$

$$\bar{C}_{xy} = \sum_{k=1}^{N} \left(\frac{\mu_y E_x}{1-\mu_x\mu_y}\right)_k t_k \tilde{z}_k = \sum_{k=1}^{N} \left(\frac{\mu_x E_y}{1-\mu_x\mu_y}\right)_k t_k \tilde{z}_k \tag{65}$$

$$\bar{K}_{xy} = \sum_{k=1}^{N} (G_{xy})_k \, t_k \tilde{z}_k \tag{66}$$

where the subscript k refers to the material and geometry of the kth layer of an N-layered shell. See figure 7. A proper choice of the reference surface can make at least one of the coupling coefficients vanish. For example, if Δ is taken as

$$\Delta = \frac{\sum_{k=1}^{N} \left(\frac{\mu_y E_x}{1 - \mu_x \mu_y} \right)_k t_k \delta_k}{\bar{E}_{xy}} \tag{67}$$

the coefficient $\bar{C}_{xy}$ vanishes and if

$$\Delta = \frac{\sum_{k=1}^{N} (G_{xy})_k \, t_k \delta_k}{\bar{G}_{xy}} \tag{68}$$

the coefficient $\bar{K}_{xy}$ vanishes.

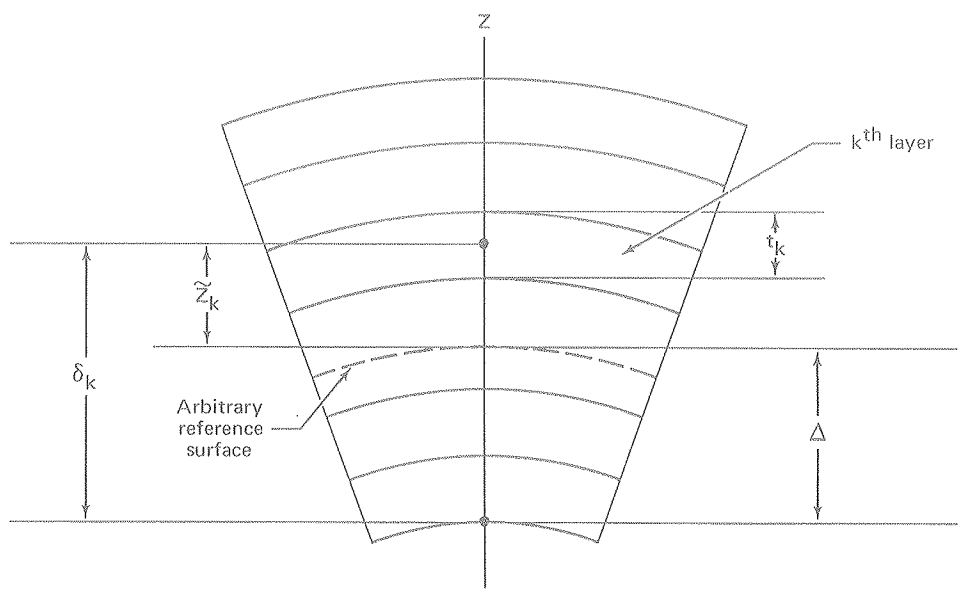

Figure 7
Multilayered orthotropic cylindrical shell geometry

4.3.6.2 Isotropic Cylinders with Stiffeners and Rings

For a cylinder consisting of a stiffened single isotropic layer and for a reference surface at the center of the layer, equations (56) to (66) reduce to

$$\bar{E}_x = \frac{Et}{1-\mu^2} + \frac{E_s A_s}{b} \tag{69}$$

$$\bar{E}_y = \frac{Et}{1-\mu^2} + \frac{E_r A_r}{d} \tag{70}$$

$$\bar{E}_{xy} = \frac{\mu Et}{1-\mu^2} \tag{71}$$

$$\bar{G}_{xy} = \frac{Et}{2(1+\mu)} \tag{72}$$

$$\bar{D}_x = \frac{Et^3}{12(1-\mu^2)} + \frac{E_s I_s}{b} + \tilde{z}_s^2 \frac{E_s A_s}{b} \tag{73}$$

$$\bar{D}_y = \frac{Et^3}{12(1-\mu^2)} + \frac{E_r I_r}{d} + \tilde{z}_r^2 \frac{E_r A_r}{d} \tag{74}$$

$$\bar{D}_{xy} = \frac{Et^3}{6(1+\mu)} + \frac{G_s J_s}{b} + \frac{G_r J_r}{d} \tag{75}$$

$$\bar{C}_x = \tilde{z}_s \frac{E_s A_s}{b} \tag{76}$$

$$\bar{C}_y = \tilde{z}_r \frac{E_r A_r}{d} \tag{77}$$

$$\bar{C}_{xy} = \bar{K}_{xy} = 0 \tag{78}$$

Appendix B

This appendix contains a C language listing of the computer code developed to evaluate the stability equations presented in NASA SP-8007. This program is general in that both longitudinal and ring stiffening effects can be accounted for when predicting critical lateral pressure. In this report, however, only stiffening ring effects were included in the SDC vacuum shell buckling analysis. The last page contains a sample dataset read by the program.

```
/*****************************************************************
 Compute Critical Buckling Loads of an Orthotropic Cylindrical Shell
 *****************************************************************/

#include  <stdio.h>

#define  MMAX  100
#define  NMAX  50
#define  PI    3.141592653589793e0

double  Cx;
double  Cxy;
double  Cy;
double  Dx;
double  Dxy;
double  Dy;
double  Ex;
double  Exy;
double  Ey;
double  Gxy;
double  Kxy;
double  L;
double  R;

double  a11;
double  a12;
double  a13;
double  a22;
double  a23;
double  a33;



/*****************/
 main (argc, argv)
/*****************/
   int   argc;
   char  **argv;
{
   input   ( );
   axial   ( );
   lateral ( );
   exit    (0);
}



/********/
 axial ()
/********/
{
   double  c0;
   double  cf;
   double  fc;
   double  Fxy();
   int     m;
   int     mm;
   int     n;
   double  nc = 1.0e+100;
   int     nn;
```

```
  double  nx;

  printf ("-------------------\n");
  printf ("Axial Buckling Load\n");
  printf ("-------------------\n\n");

  for (m=1; m<=MMAX; m++) {
     c0 = L/PI/m;
     c0 = c0*c0;

     for (n=4; n<=NMAX; n++) {

        nx = c0 * Fxy((double)m,(double)n);

        if (nx<nc) {
           nc = nx;
           fc = nc * 2.0*PI*R;
           mm = m;
           nn = n;
        }
     }
  }

  printf ("Mmax: %3d (axial)\n", MMAX);
  printf ("Nmax: %3d (circumferential)\n", NMAX);
  printf ("\n");

  cf = 0.75;

  printf ("Correlation Coefficient..... %12.3f\n",  cf);
  printf ("Axial Harmonics............ %12d%c\n",  mm, mm==MMAX?'*':' ');
  printf ("Circumferential Harmonics... %12d%c\n",  nn, nn==NMAX?'*':' ');
  printf ("Critical Fx................ %12.4e  (%.4e)\n",   cf*fc, fc);
  printf ("Critical Nx................ %12.4e  (%.4e)\n\n", cf*nc, nc);
}


/**********/
 lateral ()
/**********/
{
  double  cf;
  double  Fxy();
  int     n;
  int     nn;
  double  p;
  double  pc = 1.0e+100;

  printf ("-------------------------\n");
  printf ("Lateral Buckling Pressure\n");
  printf ("-------------------------\n\n");

  for (n=2; n<=NMAX; n++) {

     p = R * Fxy(1.0,(double)n) /n/n;

     if (p<pc) {
        pc = p;
        nn = n;
     }
```

```
   }

   cf = 0.75;

   printf ("Correlation Coefficient..... %12.3f\n",  cf);
   printf ("Circumferential Harmonics... %12d%c\n",  nn, nn==NMAX?'*':' ');
   printf ("Critical P................. %12.4e  (%.4e)\n\n", cf*pc, pc);
}



/****************/
 double Fxy (m,n)
/****************/
   double  m, n;
{
   double  c1;
   double  c2;
   double  m1;
   double  m12;
   double  m13;
   double  m14;
   double  nr;
   double  nr2;
   double  nr3;
   double  nr4;

   m1  = m*PI/L;
   m12 = m1  * m1;
   m13 = m1  * m12;
   m14 = m12 * m12;

   nr  = n/R;
   nr2 = nr  * nr;
   nr3 = nr  * nr2;
   nr4 = nr2 * nr2;

   a11 = Ex*m12 + Gxy*nr2;
   a22 = Ey*nr2 + Gxy*m12;
   a33 = Dx*m14 + Dxy*m12*nr2 + Dy*nr4 + Ey/R/R + 2.0*(Cy*nr2 + Cxy*m12)/R;

   a12 = (Exy +     Gxy)*m1*nr;
   a13 = (Cxy + 2.0*Kxy)*m1*nr2 + Exy*m1/R + Cx*m13;
   a23 = (Cxy + 2.0*Kxy)*m12*nr + Ey *nr/R + Cy*nr3;

/**********************************
   printf ("a11: %14e\n", *a11);
   printf ("a22: %14e\n", *a22);
   printf ("a33: %14e\n", *a33);
   printf ("a12: %14e\n", *a12);
   printf ("a13: %14e\n", *a13);
   printf ("a23: %14e\n", *a23);
**********************************/

   c1 = a11 * (a22*a33 - a23*a23)
      - a12 * (a12*a33 - a13*a23)
      + a13 * (a12*a23 - a13*a22);

   c2 = a11*a22 - a12*a12;
```

```
/**********************************
   printf ("c1: %14e\n", c1);
   printf ("c2: %14e\n", c2);
***********************************/

   return( c1/c2 );
}



/*********/
 input ()
/********/
{
   double  A;
   double  Ar;
   double  As;
   double  Br;
   double  Bs;
   double  E;
   double  E1;
   double  E2;
   double  E3;
   double  Er;
   double  Es;
   double  Gr;
   double  Gs;
   double  I;
   double  Ir;
   double  Is;
   double  Jr;
   double  Js;
   double  nu;
   double  Zr;
   double  Zs;

   double  hr;
   double  tr;
   double  wr;

   scanf ("%E%*[^\n]", &A );
   scanf ("%E%*[^\n]", &E );
   scanf ("%E%*[^\n]", &I );
   scanf ("%E%*[^\n]", &L );
   scanf ("%E%*[^\n]", &nu);
   scanf ("%E%*[^\n]", &R );

   scanf ("%E%*[^\n]", &As);
   scanf ("%E%*[^\n]", &Bs);
   scanf ("%E%*[^\n]", &Es);
   scanf ("%E%*[^\n]", &Gs);
   scanf ("%E%*[^\n]", &Is);
   scanf ("%E%*[^\n]", &Js);
   scanf ("%E%*[^\n]", &Zs);

   scanf ("%E%*[^\n]", &Ar);
   scanf ("%E%*[^\n]", &Br);
   scanf ("%E%*[^\n]", &Er);
   scanf ("%E%*[^\n]", &Gr);
   scanf ("%E%*[^\n]", &Ir);
```

```
scanf ("%E%*[^\n]", &Jr);
scanf ("%E%*[^\n]", &Zr);

if (Ar==0.0) {
   scanf ("%E%*[^\n]", &hr);
   scanf ("%E%*[^\n]", &tr);
   scanf ("%E%*[^\n]", &wr);
}

printf ("-------------------------------------------------------\n");
printf ("Input Parameters for an Orthotropic Cylindrical Shell\n");
printf ("-------------------------------------------------------\n\n");
printf ("L.... %12.4e\n", L );
printf ("R.... %12.4e\n", R );
printf ("\n");

printf ("A.... %12.4e\n", A );
printf ("E.... %12.4e\n", E );
printf ("I.... %12.4e\n", I );
printf ("Nu... %12.4e\n", nu);
printf ("\n");

printf ("As... %12.4e\n", As);
printf ("Bs... %12.4e\n", Bs);
printf ("Es... %12.4e\n", Es);
printf ("Gs... %12.4e\n", Gs);
printf ("Is... %12.4e  Is,ref: %.4e\n", Is, Is+Zs*Zs*As);
printf ("Js... %12.4e\n", Js);
printf ("Zs... %12.4e\n", Zs);
printf ("\n");

if (Ar==0.0) {

   /* Single Flange Ring
   -------------------*/
   printf ("Single Flange Ring\n");
   printf ("------------------\n");
   printf ("hr... %12.4e\n", hr);
   printf ("tr... %12.4e\n", tr);
   printf ("wr... %12.4e\n", wr);
   printf ("\n");

   Ar = wr*tr;
   Ir = Ar*tr*tr/12.0;
   Jr = 0.0;

   if (hr>0.0)
      Zr = hr - 0.5*tr;
   else
      Zr = hr + 0.5*tr;
}

printf ("Ar... %12.4e\n", Ar);
printf ("Br... %12.4e\n", Br);
printf ("Er... %12.4e\n", Er);
printf ("Gr... %12.4e\n", Gr);
printf ("Ir... %12.4e  Ir,ref: %.4e\n", Ir, Ir+Zr*Zr*Ar);
printf ("Jr... %12.4e\n", Jr);
printf ("Zr... %12.4e\n", Zr);
printf ("\n");
```

```
  E1 =     E*A/(1.0-nu*nu);
  E2 =     E*I/(1.0-nu*nu);
  E3 = 2.0*E*I/(1.0+nu);

  Ex  = E1 + Es*As/Bs;
  Ey  = E1 + Er*Ar/Br;
  Exy = E1 * nu;

  Cx  = Es*As*Zs/Bs;
  Cy  = Er*Ar*Zr/Br;
  Cxy = 0.0;

  Dx  = E2 + Es*Is/Bs + Cx*Zs;
  Dy  = E2 + Er*Ir/Br + Cy*Zr;
  Dxy = E3 + Gs*Js/Bs + Gr*Jr/Br;

  Gxy = E*A/2.0/(1.0+nu);
  Kxy = 0.0;

  printf ("-------------------------------------------------------\n");
  printf ("Computed Parameters for an Orthotropic Cylindrical Shell\n");
  printf ("-------------------------------------------------------\n\n");
  printf ("Ex.... %12.4e\n", Ex );
  printf ("Ey.... %12.4e\n", Ey );
  printf ("Exy... %12.4e\n", Exy);
  printf ("\n");

  printf ("Cx.... %12.4e\n", Cx );
  printf ("Cy.... %12.4e\n", Cy );
  printf ("Cxy... %12.4e\n", Cxy);
  printf ("\n");

  printf ("Dx...  %12.4e\n", Dx );
  printf ("Dy...  %12.4e\n", Dy );
  printf ("Dxy... %12.4e\n", Dxy);
  printf ("\n");

  printf ("Gxy... %12.4e\n", Gxy);
  printf ("Kxy... %12.4e\n", Kxy);
  printf ("\n");
}
```

```
0.1815          : A  -- cylinder area (effective thickness for Isogrid)
10.e+6          : E  -- cylinder Young's modulus
0.0109          : I  -- cylinder area moment of inertia
360.0           : L  -- cylinder length
0.33            : Nu -- cylinder Poisson's ratio
80.0            : R  -- cylinder radius
0.0             : As -- stiffener area
1.0             : Bs -- stiffener spacing
0.0             : Es -- stiffener Young's modulus
0.0             : Gs -- stiffener shear modulus
0.0             : Is -- stiffener centroidal moment of inertia
0.0             : Js -- stiffener beam torsion constant
0.0             : Zs -- stiffener centroidal distance from reference surface (+ outward)
0.0             : Ar -- ring area
45.0            : Br -- ring spacing
30.0e6          : Er -- ring Young's modulus
10.0e6          : Gr -- ring shear modulus
0.0             : Ir -- ring centroidal moment of inertia
0.0             : Jr -- ring beam torsion constant
0.0             : Zr -- ring centroidal distance from reference surface (+outward)
2.59            : hr -- ring height from reference surface (+ outward)
1.65            : tr -- ring flange thickness
0.76            : wr -- ring flange width
```