SDC SOLENOID DESIGN NOTE #131

TITLE: Isogrid Outer Vacuum Shell For SDC Solenoid
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ABSTRACT: This design note contains a study of an isogrid outer vacuum shell for the SDC superconducting solenoid. It includes a calculation of isogrid skin thickness t , web depth d, and web width b for different node spacing (or cell size) with straight side webs. A mapping technique is used to solve the stability equations. Finally, the radiation length behavior has been also studied.

COMPUTATIONAL METHOD: The vacuum shell is considered to be a cylinder under an uniform external pressure differential of 15 psi. The critical collapsing pressure p_{cr} enters the three stability equations given by General Dynamics "Isogrid Design Handbook"(1)

$$N_{cr}(\text{general instability}) = \frac{C_o E R}{\left(\frac{R}{t}\right)^{2.5} \cdot \left(\frac{L}{R}\right)} \cdot \frac{\beta^{1.5}}{(1+\alpha)^{0.5}} \tag{1}$$

$$N_{cr}(\text{skin buckling}) = C_1 \cdot E \cdot (1+\alpha) \cdot \frac{t^3}{h^2} \tag{2}$$

$$N_{cr}(\text{rib crippling}) = C_2 \cdot E \cdot (1+\alpha) \cdot \left(\frac{b}{d}\right)^2 \cdot t \tag{3}$$

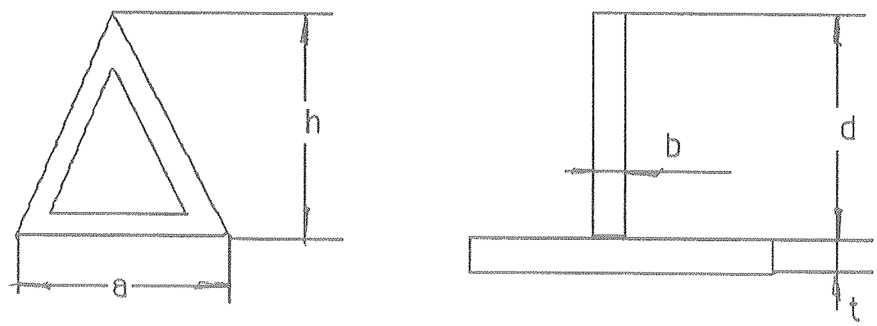
where the C_o=0.702, C_1=6.80,and C_2=0.616. The parameter N_{cr}, α, δ and β are defined as following

$$\alpha = \frac{b \cdot d}{t \cdot h} \qquad\qquad \delta = \frac{d}{t}$$

$$\beta^2 = \left[3\alpha(1+\delta)^2 + (1+\alpha)\cdot\left(1+\alpha\delta^2\right)\right]$$

$$N_{cr} = p_{cr} \cdot R$$

where E=modulus, and R= outer radius of shell. We use an engineering standard (Fermilab E.S.+H Manual, chapter 5033) which requires that p_{cr}=30 psi.



For the SDC solenoid, the overall isogrid thickness was arbitrarily chosen to be 2". There is accordingly one constraint equation:

$$S = (t+d) = 2" \qquad (4)$$

There are now four equations and three unknowns. We solve equation (1), (2), and (4) simultaneously, for a given value of h, for b,d, and t and show that these values satisfy equation(3). We begin starting with equation (4),

$$S = (t + d) = (t + \delta t) = t\ (1 + \delta) \qquad (5)$$

and solving for t

$$t = \frac{S}{1+\delta} \qquad (6)$$

Substituting equation (6) into the equation (1) for t, we get

$$N_{cr} = \frac{C_o ER}{\left(\frac{\frac{R}{S}}{1 + \delta}\right)^{2.5} \cdot \left(\frac{L}{R}\right)} \cdot \frac{\beta^{1.5}}{(1 + \alpha)^{0.5}}$$

$$= \frac{C_o ER}{\left(\frac{R}{S}\right)^{2.5} \cdot \left(\frac{L}{R}\right)} \cdot \frac{\beta^{1.5}}{(1 + \alpha)^{0.5}(1 + \delta)^{2.5}} \qquad (7)$$

and rearranging it

$$\frac{N_{cr} \cdot \left(\frac{R}{S}\right)^{2.5} \cdot \left(\frac{L}{R}\right)}{C_o \cdot E \cdot R} = \frac{\beta^{1.5}}{(1 + \alpha)^{0.5}(1 + \delta)^{2.5}} \qquad (8)$$

By substituting equation (6) into equation (2) for t, we get

$$\frac{Ncr \cdot h^2}{E \cdot C_1 \cdot S^3} = \frac{1 + \alpha}{\left(1 + \delta^3\right)} \qquad (9)$$

If we now define two variables

$$X = \frac{1 + \alpha}{\left(1 + \delta^3\right)} \qquad (10)$$

$$Y = \frac{\beta^{1.5}}{(1 + \alpha)^{0.5}(1 + \delta)^{2.5}} \qquad (11)$$

and substitute them into equation (9) and (10), then

$$X = \frac{Ncr \cdot h^2}{E \cdot C_1 \cdot S^3} \qquad (12)$$

$$Y = \frac{N_{cr} \cdot \left(\frac{R}{S}\right)^{2.5} \cdot \left(\frac{L}{R}\right)}{C_o \cdot E \cdot R} \quad (13)$$

Since S, h, and N_{cr} are given, the X and Y can be computed from equation (12) and (13). To solve equation (10),and (11) for a given X-Y , we map α and δ into X-Y coordinate by a mapping transformation (shown in Appendix B). Values of α and δ can be immediately found from this chart and therefore t, d and b can be calculated. Finally, a checking procedure is used to insure that the dimensions calculated this way satisfy the three stability equations. This is done by substituting t, d, and d back into equation (1), (2), and (3) and solving for p_{cr}.

CALCULATION RESULT: Table 1 shows the calculation result for the six node spacing from 4" to 15". It is found that the skin thickness t decreases as the cell size. However, the smaller cell size are less efficient with respect to the weight. A 12" cell size gives the minimum weight in terms of a minimum average thickness (see Appendix C for the calculation of the average thickness). The 4" cell size gives the largest value of average thickness for those cell sizes studied. It is also interested to notice that the solution given by this method is not a optimum value. The Figure 1 shows a study of the a 4" cell . Point A is a solution given by this method; the other points are obtained by changing the t and b around the solution neighborhood to satisfy the stability equations. The minimum weight solution seems to be around b=0.25-0.3". This implies that the one iteration may not be enough. It is also interesting to learn form this curve that if the width of the web, b, goes to zero, the curve gives a 1" average thickness, which is a solution for an aluminum solid plate.

The two-dimensional figures in Appendix A show the outer shell thickness, in radiation lengths, as a function of pseudorapidity for several isogrid cell sizes. The sketch in Figure A-1 shows the plane in which the thickness was calculated. In general, the radiation thickness is characterized by a series of peaks rising from a smoothly increasing base line value. These peaks correspond to the walls of the cells. It can be seen from the figures that the maximum value of the peaks is between 0.6 and 0.7 radiation lengths. The peaks for large cells, Figure A-5, are wider, i.e. subtend a large angle, and occur less frequently than those for smaller cells, Figure A-1. For large cells the peaks rise from the smooth curve for the skin thickness alone, which increases with angle, out to 1.4 psedorapidity. For small cells the peaks return to the skin curve out about 0.9 in pseudorapidity, after which the skin is shadowed by the adjacent cell walls and base curve rises as the ray intercepts multiple walls.

REFERENCES:

1. "Isogrid Design Handbook" , General Dynamics, 1973

TABLE 1

CELL SIZE a (IN)	SKIN THICKNESS t_s (IN)	WEB WIDTH b_{WEB} (IN)	WEB DEPTH d_{WEB} (IN)	AVERAGE Thickness (IN)	P_{cr} (PSI) (General Instability)	P_{cr} (PSI) (SKIN Buckling)	P_{cr} (PSI) (WEB)	τ_{max} (PSI) (hoop)	D. BENDING STIFF.
15"	0.14	1.2	1.86	0.547	31.72	30.67	9956	7725	1.36×10^6
12"	0.12	0.95	1.88	0.526	30.19	32.95	5717	825.3	1.33×10^6
10"	0.10	0.85	1.9	0.5387	30.02	32.34	4398	8408	1.33×10^6
8"	0.08	0.73	1.92	0.5537	29.58	31.877	3130	8533	1.31×10^6
6"	0.06	0.62	1.94	0.5972	29.96	32.91	2283	8264	1.32×10^6
4"	0.038	0.47	1.962	0.6493	29.98	31.00	1339	7918	1.30×10^6

The Average Thickness As a Function of Web Width

Fig 1 For Cell Size = 4"

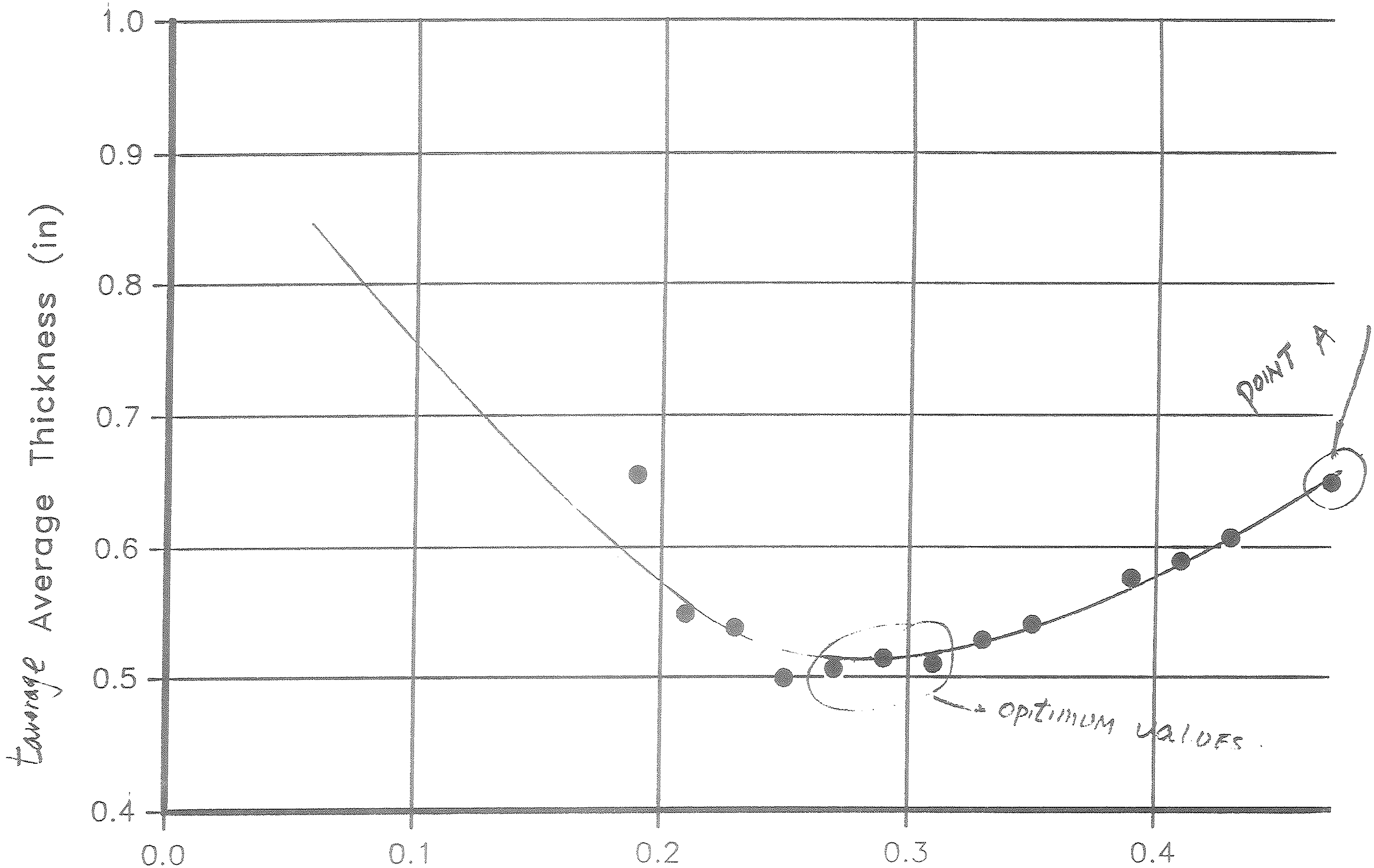


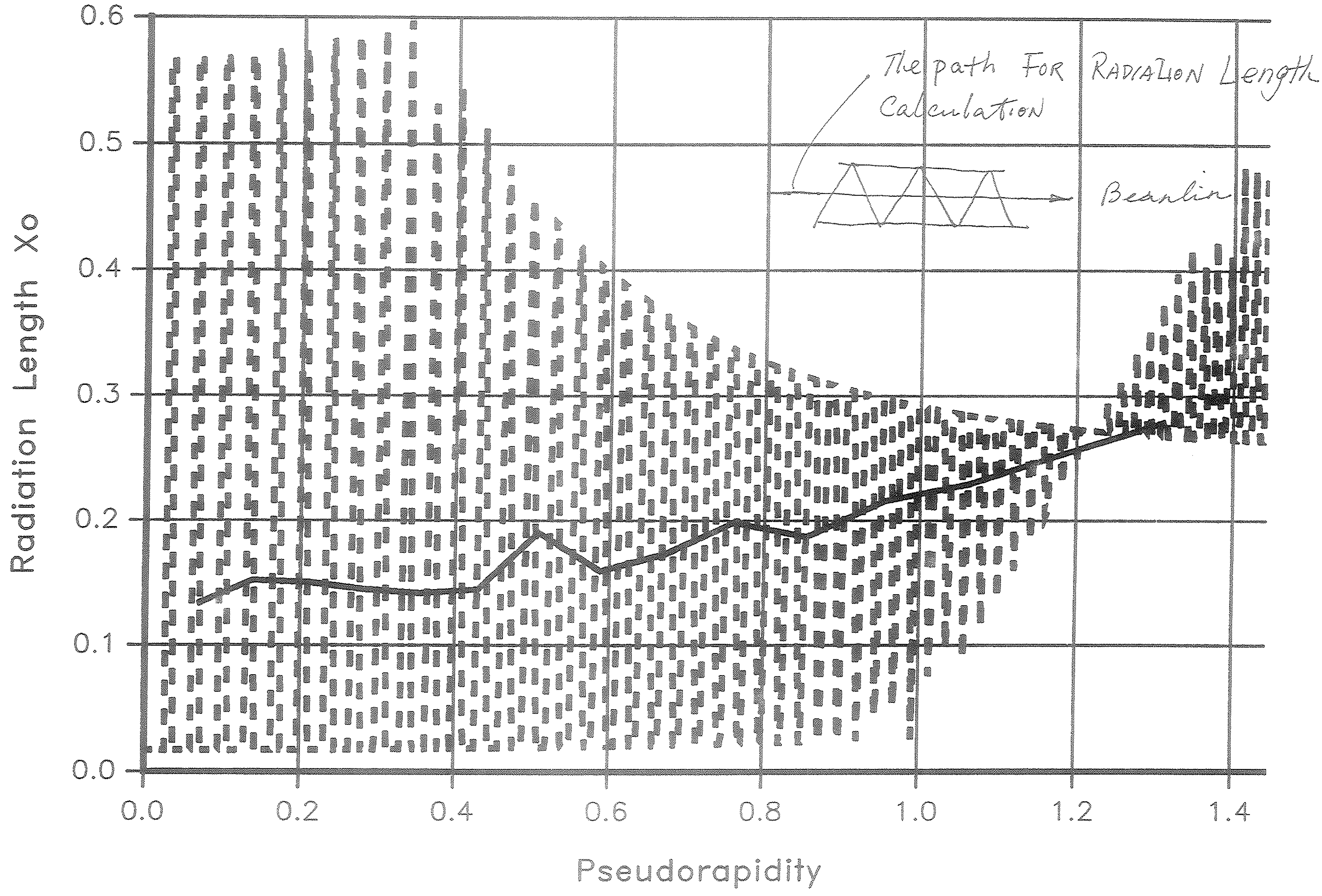
FIG 1.1 The Width of Web (in) (b)

APPENDIX A

(RADIATION LENGTH)

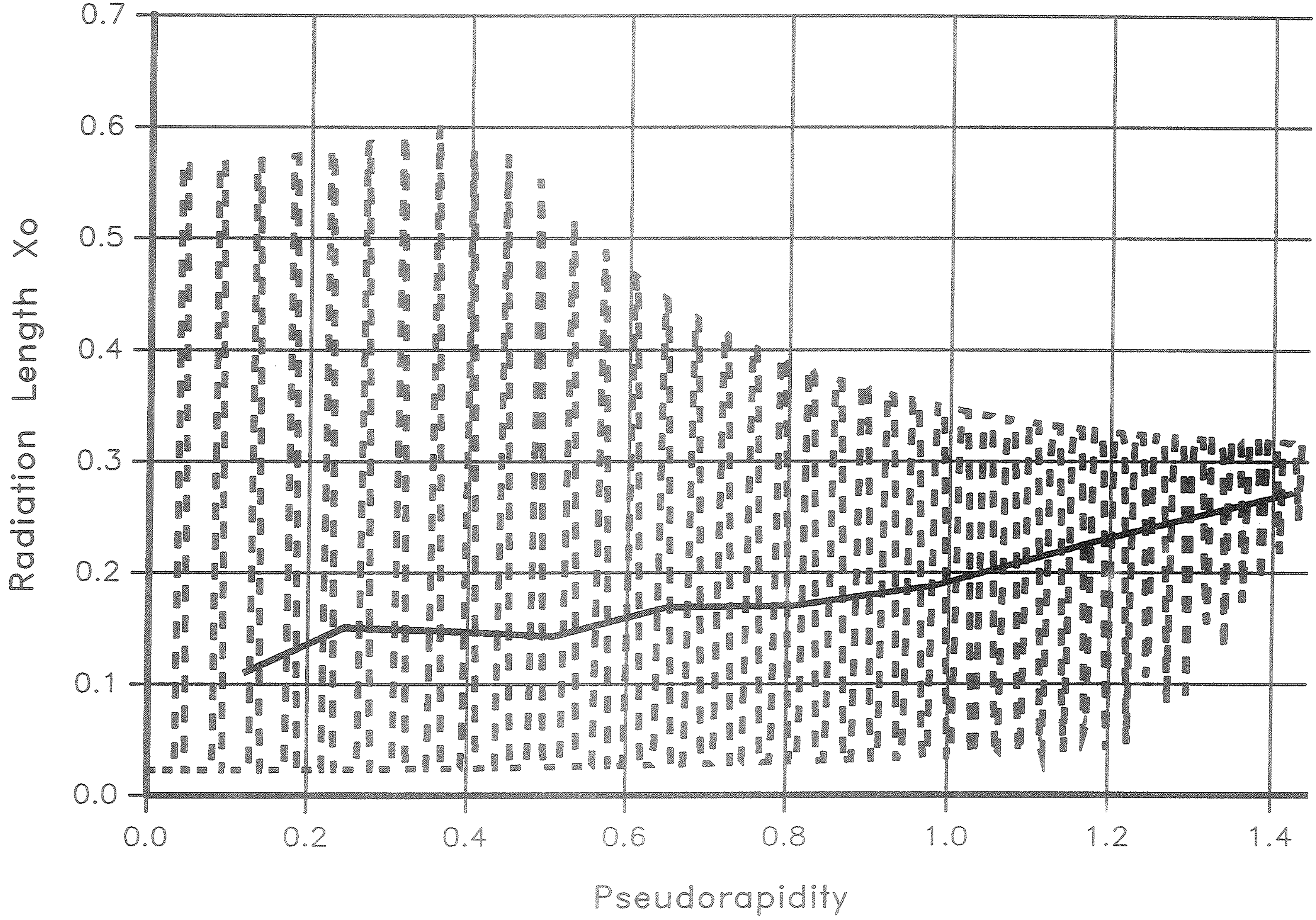
Radiation Length VS the Pseudorapidity

Fig. A-1 For cell size (a=6")



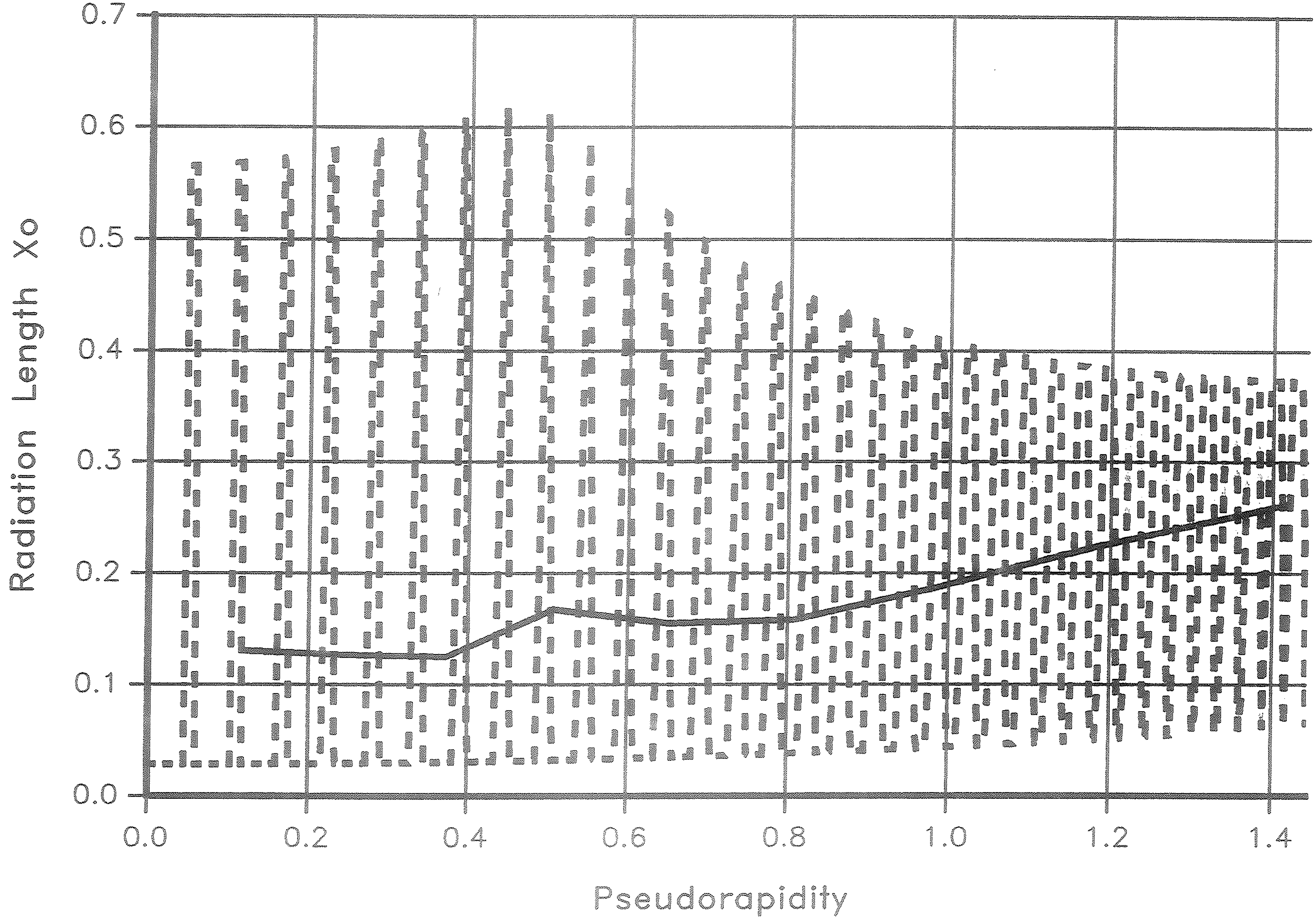
Radiation Length VS the Pseudorapidity

Fig A-2   For cell size (a=8")



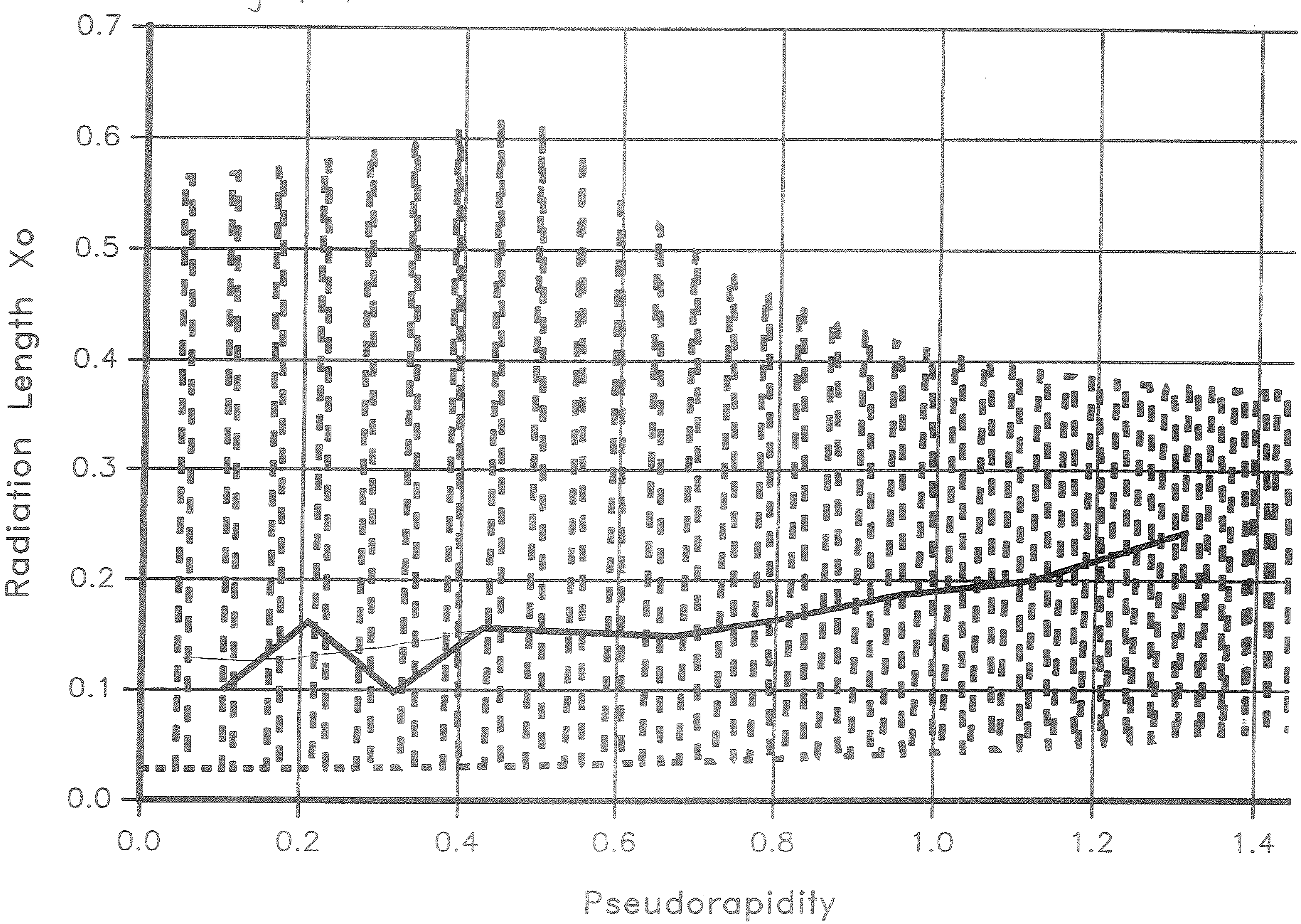
Radiation Length VS the Pseudorapidity

Fig. A-3 For cell size (a=10")



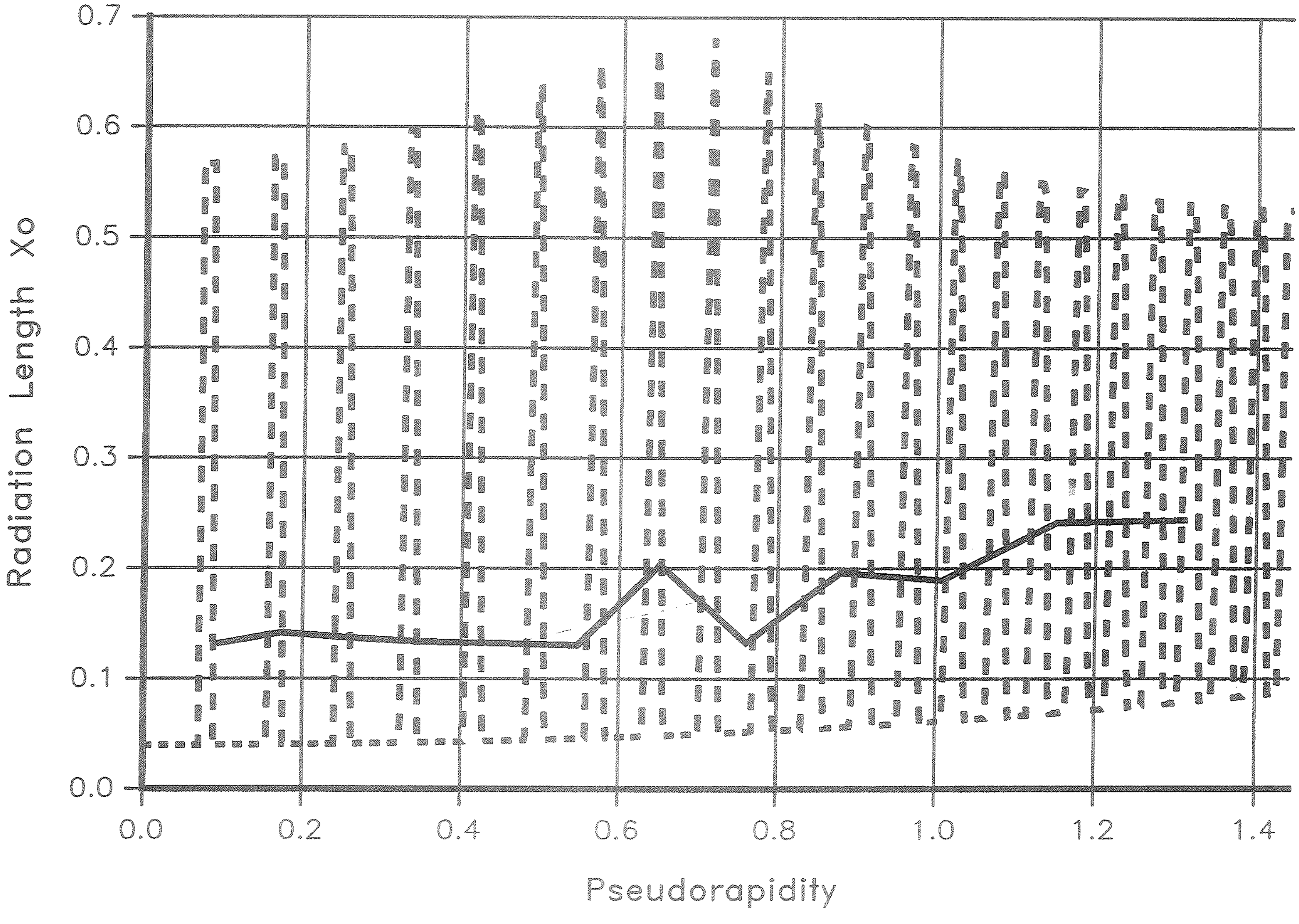
Radiation Length VS the Pseudorapidity

Fig A-4 For cell size (a=12")



Radiation Length VS the Pseudorapidity

Fig A-5 For cell size (a=15")

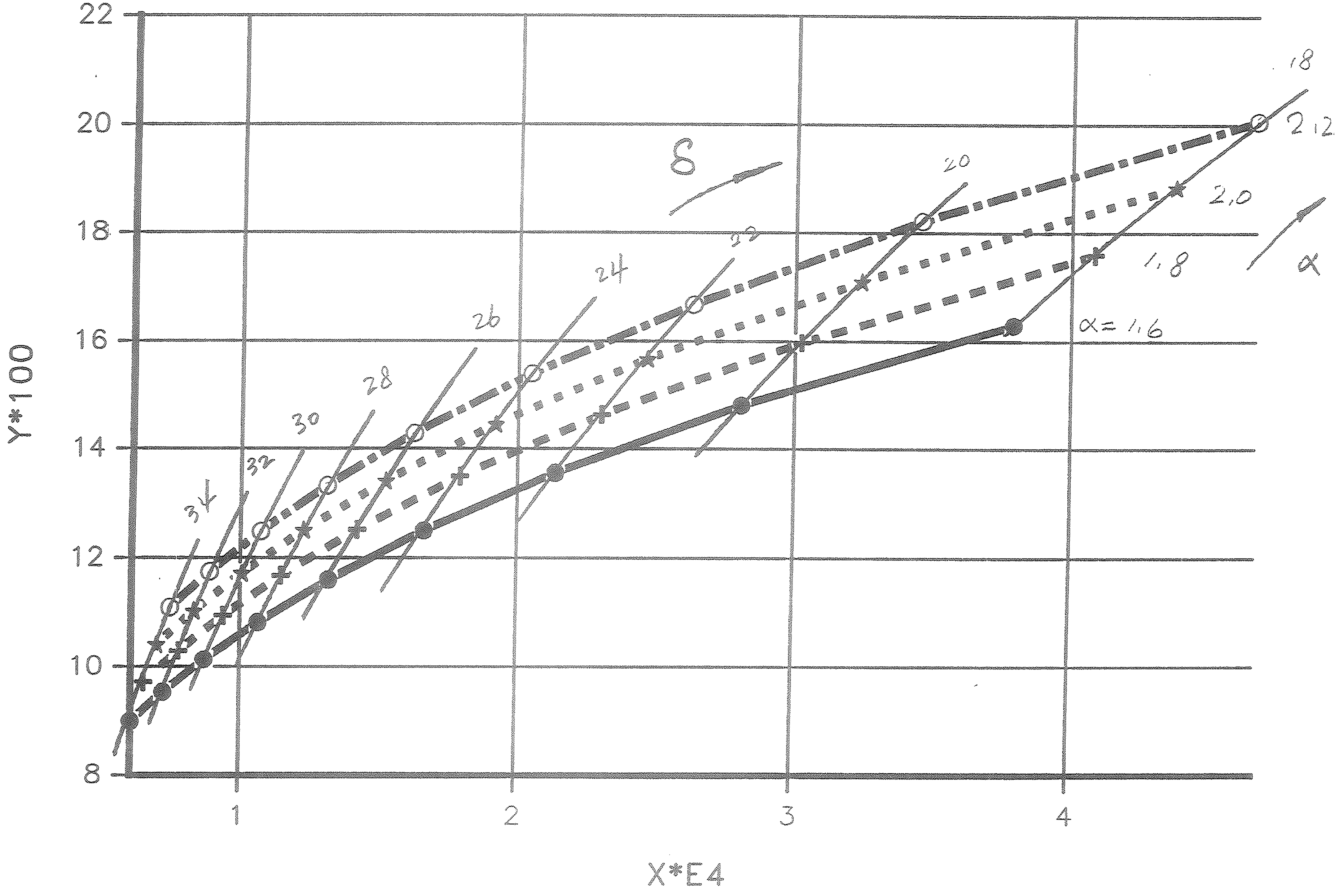


APPENDIX B

($\alpha - \delta \Leftrightarrow X - Y$) Chart

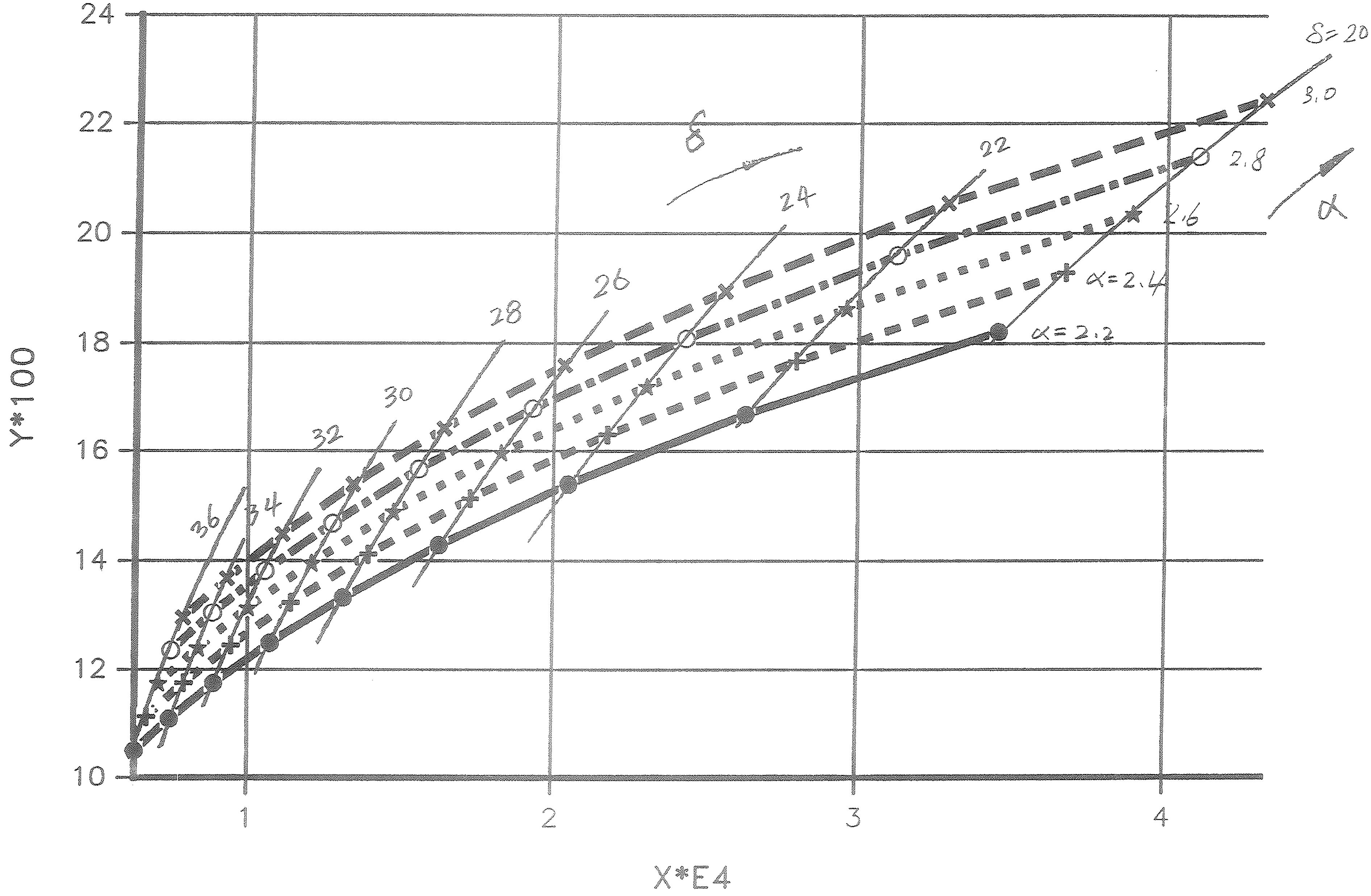
X–Y and α – δ Relation

α = 1.6, 1.8, 2.0, 2.2 and δ = 18, 20, 22, 24, 28, 30, 32, 34



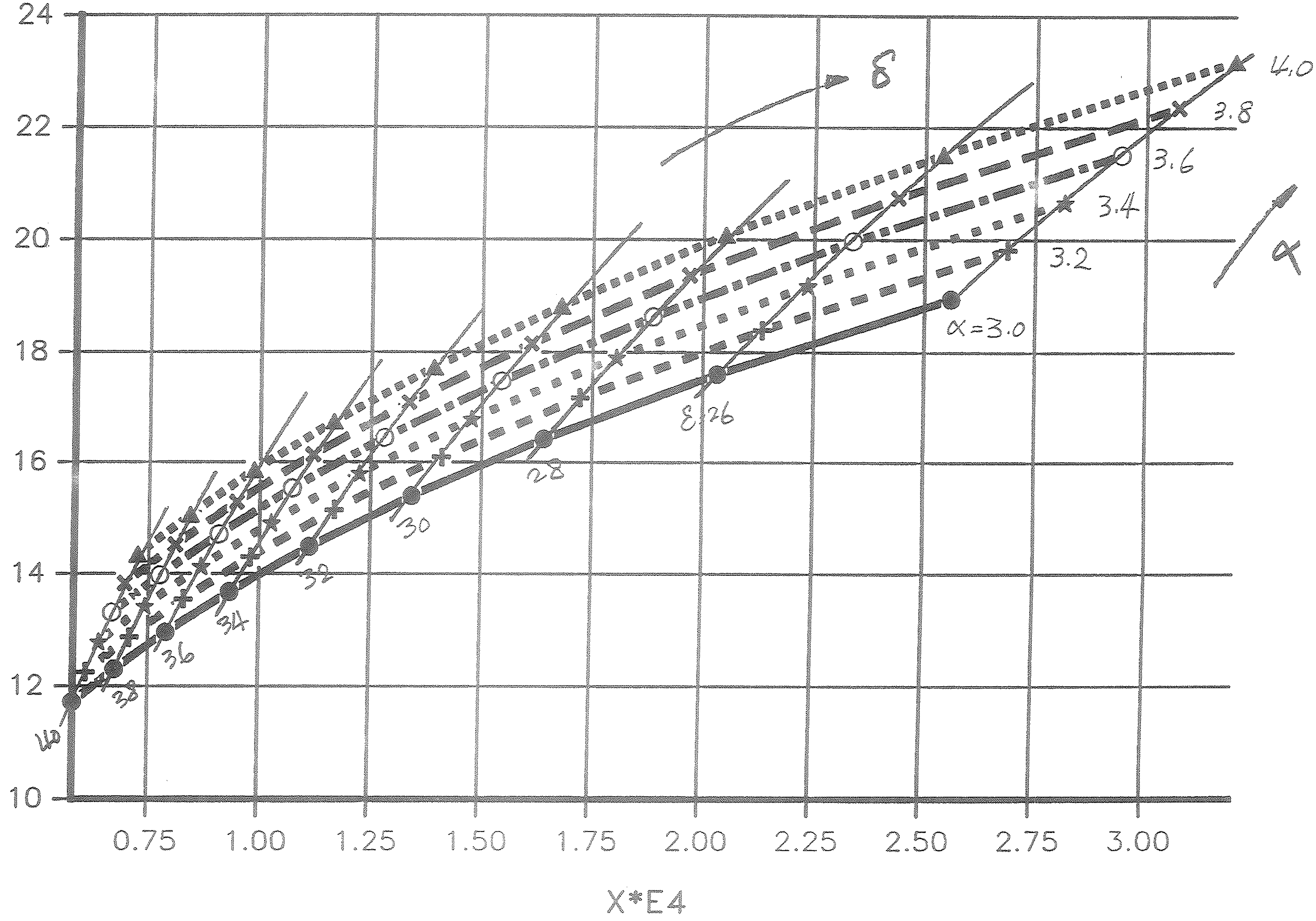
X—Y and α — δ Relation

α = 2.2,2.4,2.6,2.8,3.0 and δ = 20,22,24,26,28,30,32,34,36



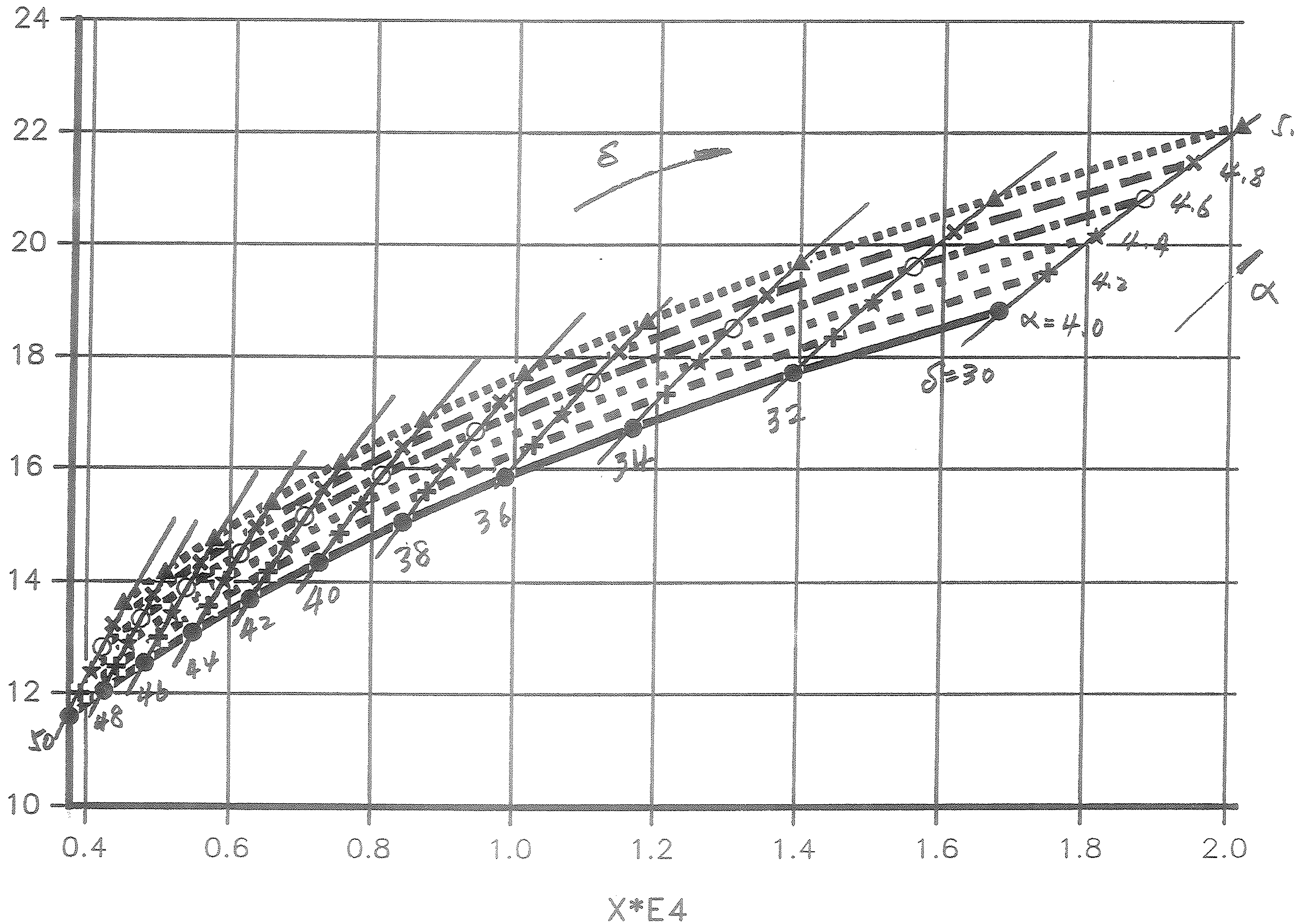
X–Y and $\alpha - \delta$ Relation

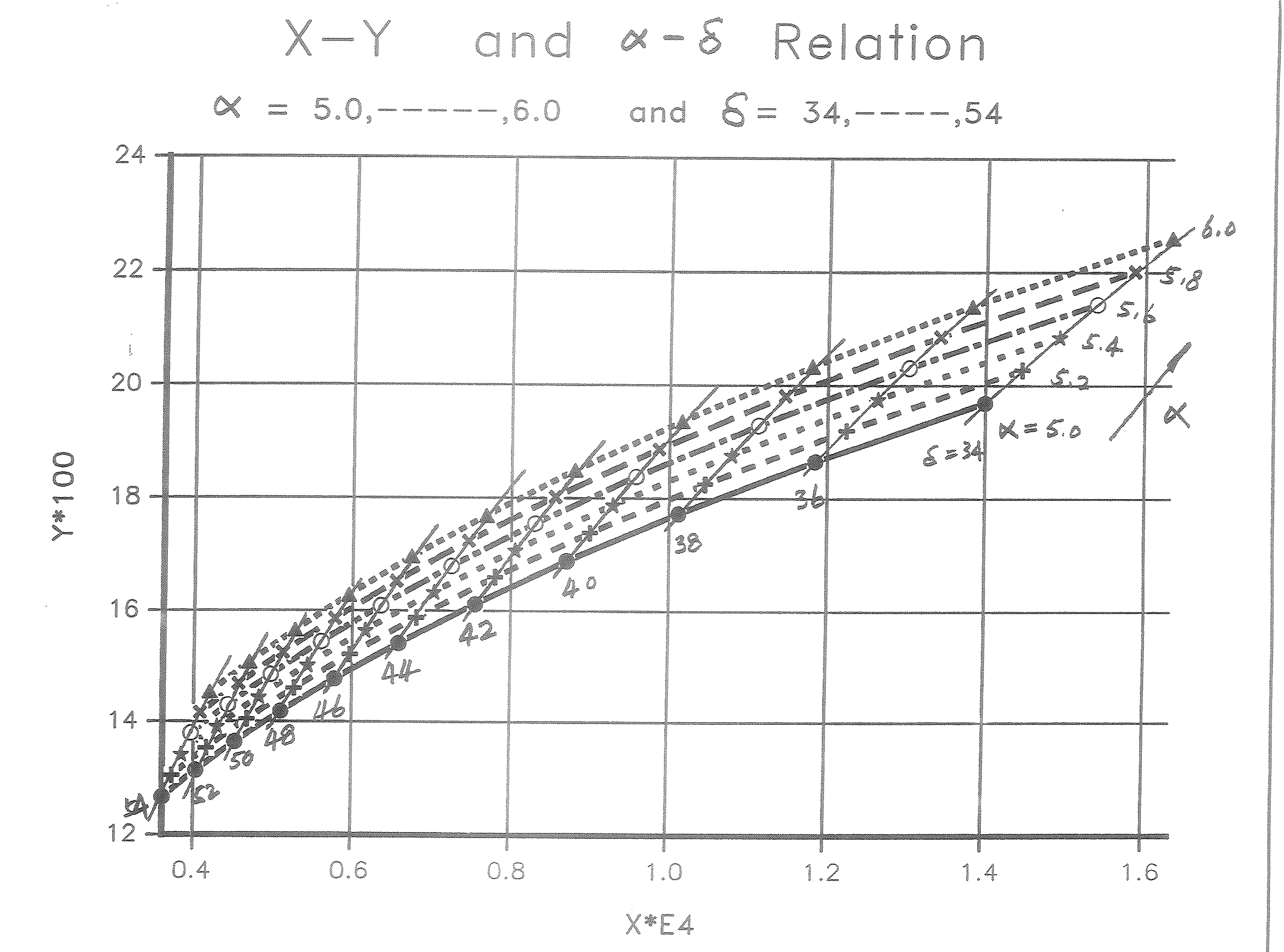
α = 3.0,3.2,3.4,3.6,3.8,4.0 and δ = 24,26,28,30,32,34,36,38,40



X–Y and α – δ Relation

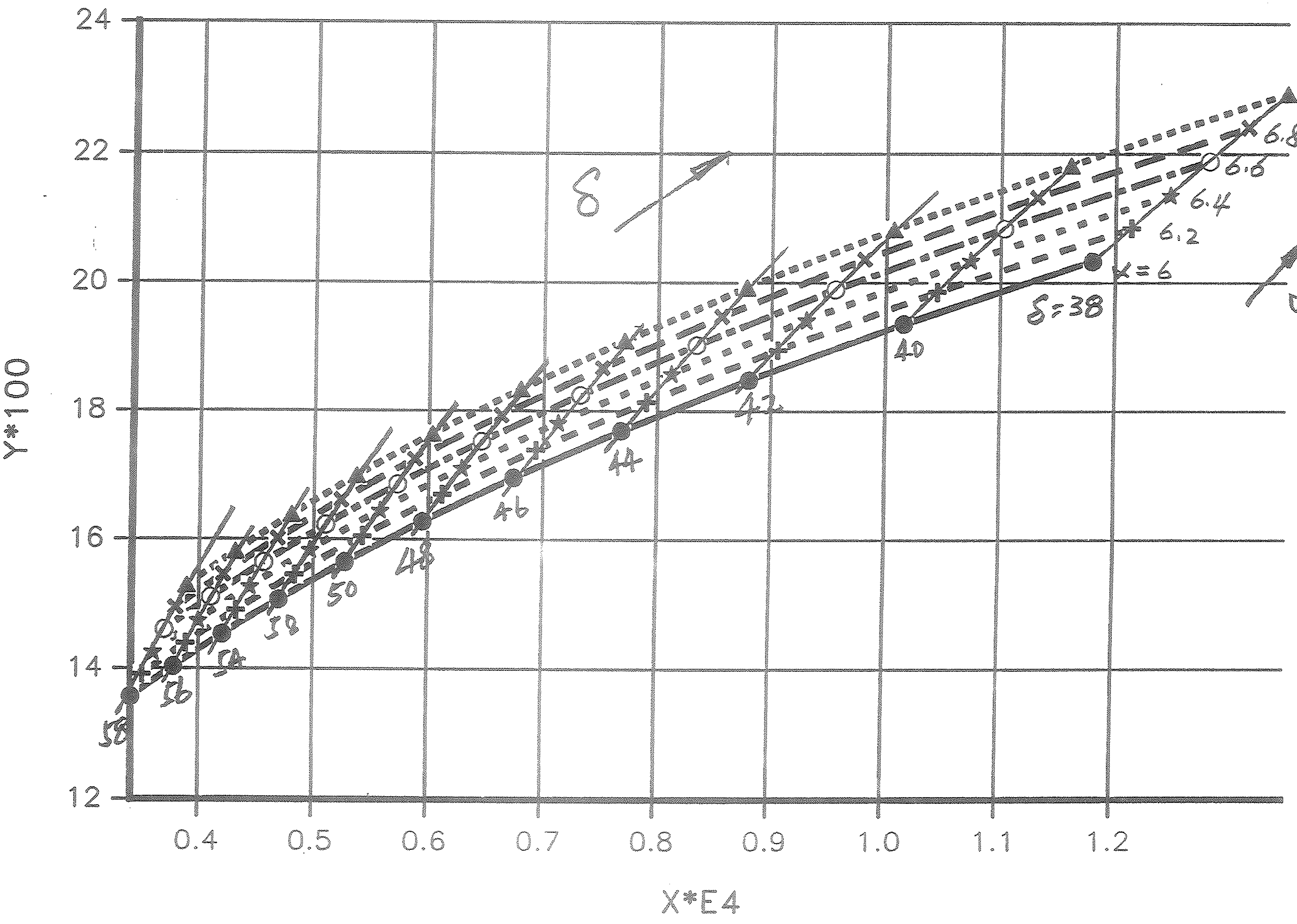
α = 4.0,4.2,4.4,4.6,4.8,5.0 and δ = 30-------50




X-Y and α-δ Relation
α = 5.0,-----,6.0 and δ= 34,----,54
Y*100
X*E4
24
22
20
18
16
14
12
0.4
0.6
0.8
1.0
1.2
1.4
1.6
6.0
5.8
5.6
5.4
5.2
α=5.0
α
δ=34
36
38
40
42
44
46
48
50
52
54

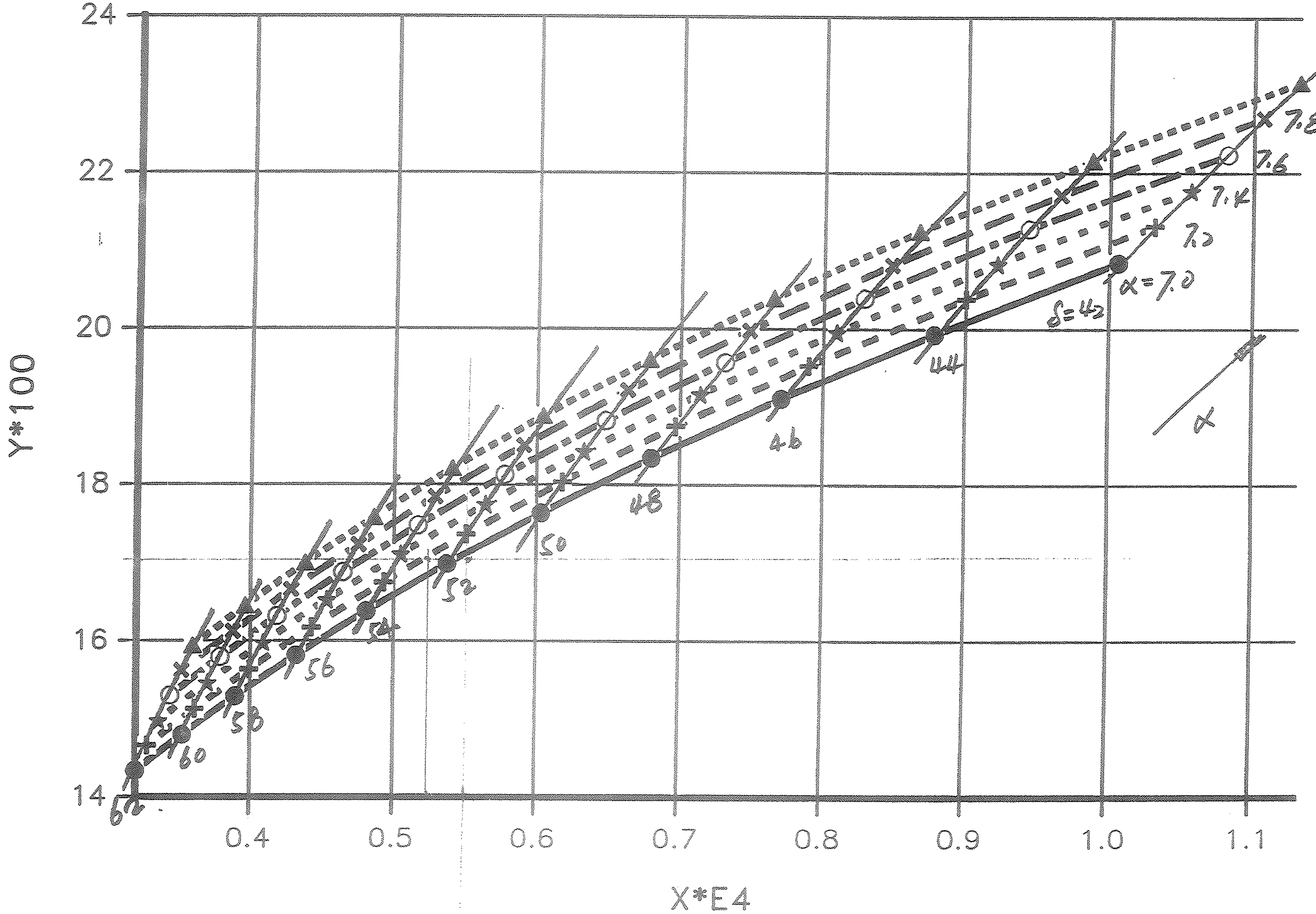
X–Y and α-δ Relation

α = 6.0----, 7.0 and δ = 38,----,58



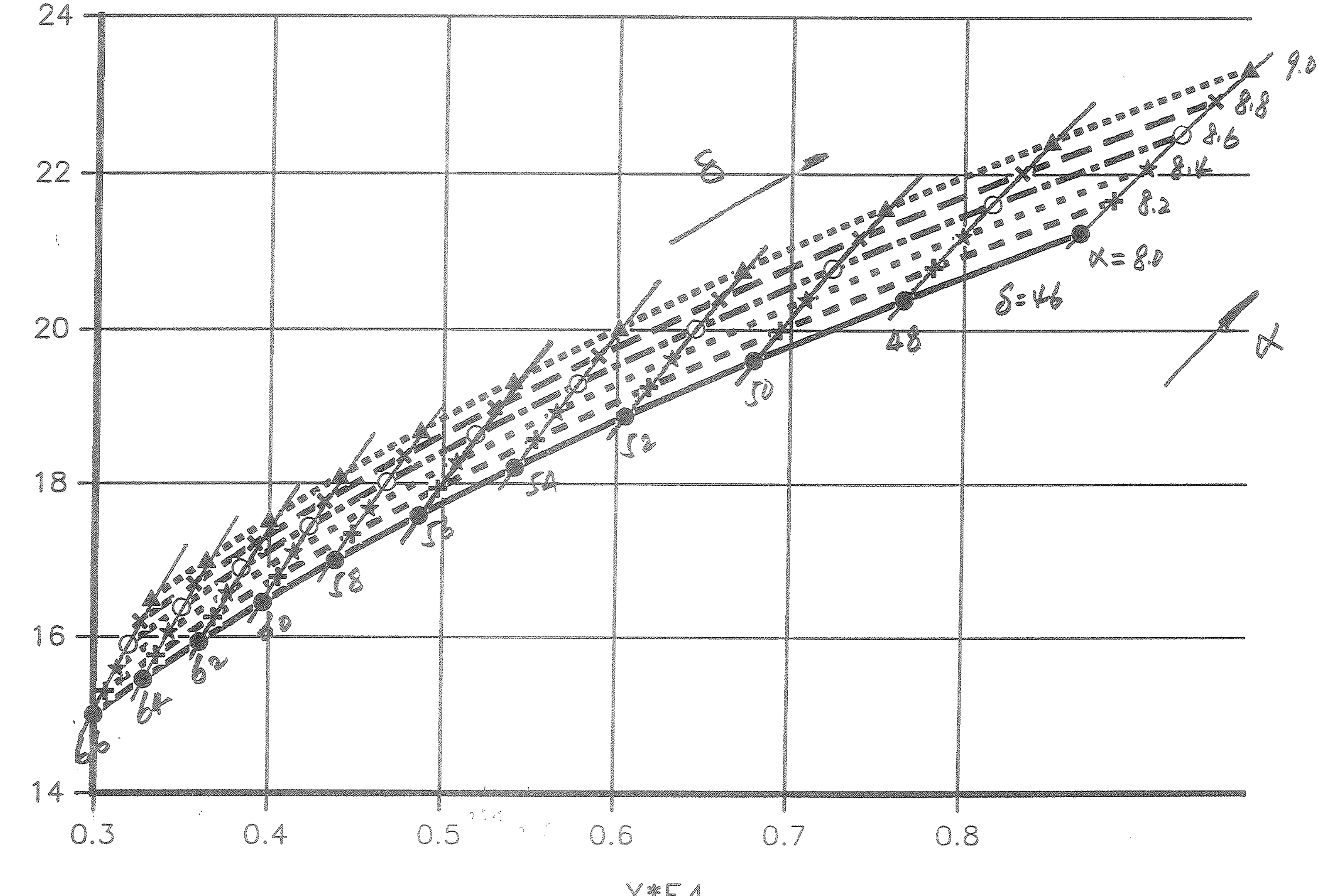
X-Y and α-δ Relation

α = 7.0,---,8.0, and δ = 42,-----,62



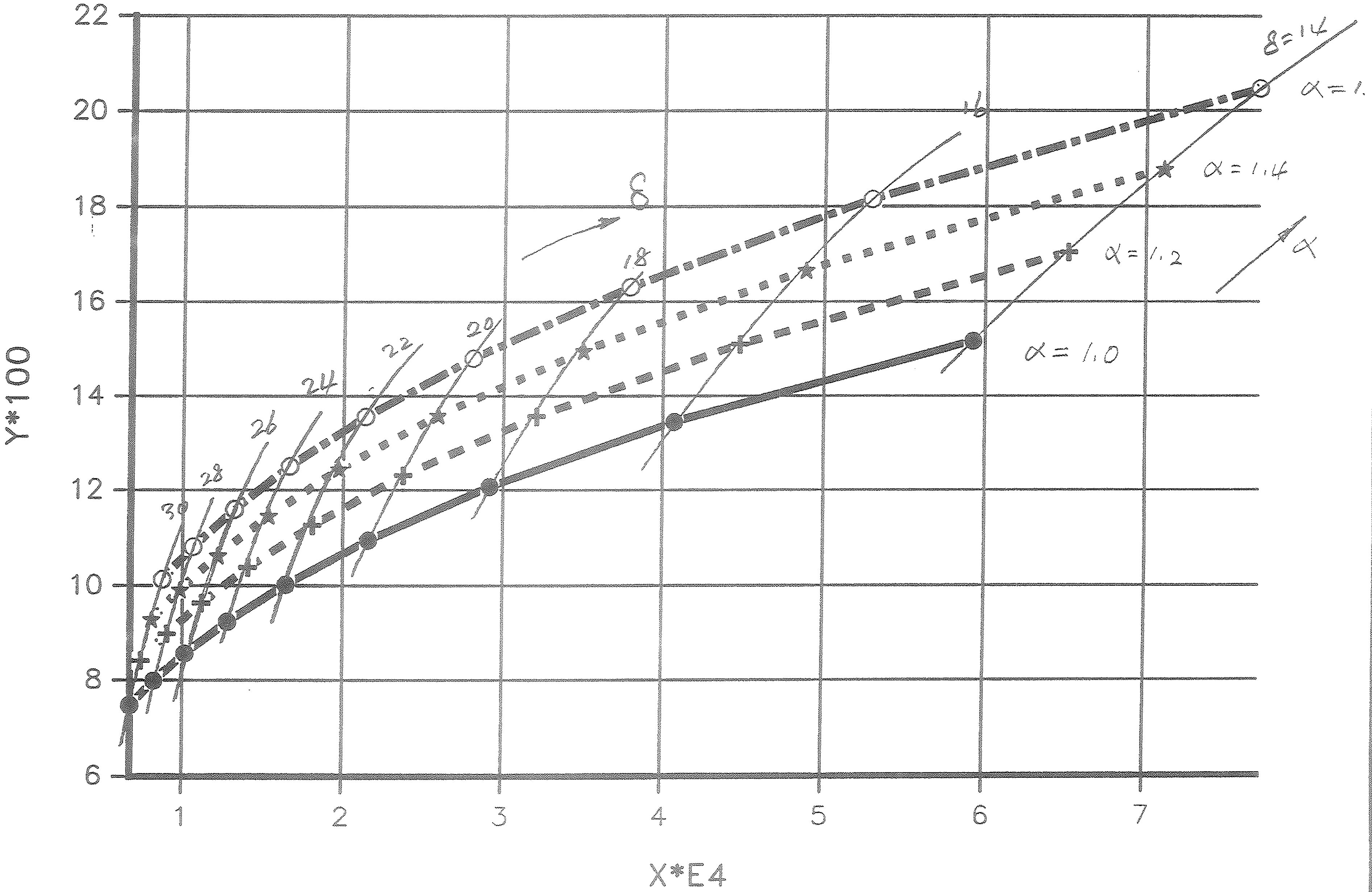
X–Y and α–δ Relation

α = 8.0,----,9.0 and δ = 46,-----,66



X–Y AND α – δ RELATION

$\alpha = 1.0, ----, 1.6$ AND $\delta = 14, ---, 30$



APPENDIX C

(AVERAGE THICKNESS)

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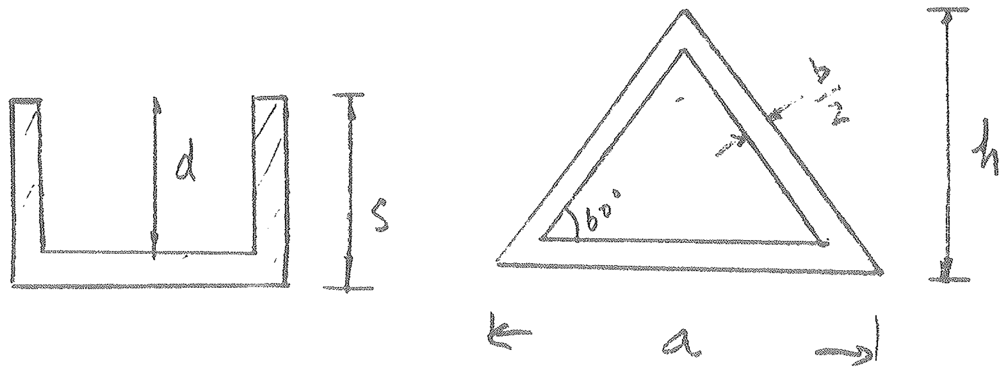
ENGINEERING NOTE

SECTION	PROJECT	SERIAL-CATEGORY	PAGE

SUBJECT: Calculation For Average Thickness

NAME	
DATE	REVISION DATE



Total weight of a cell.

$$W = (V_{big} - V_{small})\,\rho$$

$$= A_{big} \cdot t_{average} \cdot \rho$$

$$V_{big} = \frac{1}{2} a \cdot h \cdot S$$

$$V_{small} = \frac{1}{2}\left(a - 2\frac{b}{2\cos 30^\circ}\right)\left(h - \frac{b}{2} - \frac{b}{2\sin 30^\circ}\right) \cdot d$$

$$A_{big} = \frac{1}{2} \cdot a \cdot h$$

$$\therefore \quad t_{avera} = \frac{\frac{1}{2} a \cdot h \cdot S - \frac{1}{2}\left(a - 2\frac{b}{2\cos 30^\circ}\right)\left(h - \frac{b}{2} - \frac{b}{2\sin 30^\circ}\right) \cdot d}{\frac{1}{2} \cdot a \cdot h}$$