SSC DETECTOR SOLENOID DESIGN NOTE #29

TITLE: Maximum Hot Spot Temperature for the CDF Conductor: Adiabatic Heat Balance Analysis

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SUMMARY: The adiabatic hot spot temperature for an all aluminum conductor with RRR = 1500 and 1885 was calculated. (All aluminum means that the heat was generated by all the current being in the aluminum and that all the heat was absorbed by the aluminum.) It was also calculated for the CDF aluminum-copper composite conductor. The hot spot temperature calculated from the coil inductance and fast dump resistance is several times higher than that calculated from the measured quench current decay.

INTRODUCTION

From Martin Wilson's book, "Superconducting Magnets", page 201: The heat balance per unit volume of coil is

Heat generated = Heat absorbed

$$J^2(T)\ \rho(\theta)\ dT = \gamma\ C(\theta)\ d\theta \tag{1}$$

where T = time (s), θ = temperature (K), J = current density (A/m^2), ρ = resistivity (Ω-m), γ = density (kg/m^3), $C(\theta)$ = specific heat (J/kg-K). An adiabatic analysis ignores the heat capacity of the liquid or gas helium in order to give a worst-case estimate of the hot spot temperature.

If the conductor has i components in parallel

$$\Sigma\ J^2_i(T)\ \rho_i(\theta)\ dT = \Sigma\ \gamma_i\ C_i(\theta)\ d\theta \tag{2}$$

When the conductor has three components, e.g. aluminum, copper and superconductor, as the CDF conductor does, the heat balance equation becomes

$$J^2_{Cu}(T)\rho_{Cu}(\theta)dT + J^2_{Al}(T)\rho_{Al}(\theta)dT + J^2_{Sc}(T)\rho_{Sc}(\theta)dT = \gamma_{Cu}C_{Cu}(\theta)d\theta + \gamma_{Al}C_{Al}(\theta)d\theta + \gamma_{Sc}C_{Sc}(\theta)d\theta \tag{3}$$

Separating variables and integrating Eq. 1 for a single-component conductor, we get Wilson's Equation 9.4:

$$U(\theta_m) = \int_0^\infty J^2(T)\,dT = \int_{\theta_0}^{\theta_m} \gamma \frac{C(\theta)}{\rho(\theta)}\,d\theta \tag{4}$$

CALCULATE $U(\theta_m)$ FOR ALL-ALUMINUM CONDUCTOR

This calculation ignores the presence of the aluminum and the superconductor; all the current during the quench is in the aluminum and all the heat so generated is absorbed by the heat capacity of the aluminum. A later section will consider the more general case.

Figure 1 gives $U(\theta_m)$ for aluminum conductors of several RRR values. The curve for RRR ~ 500 is copied from Wilson's Fig 9.1. I calculated the curves for RRR = 1500 and 1885 using $\rho(\theta)$ and $C(\theta)$ from the "Handbook on Materials for Superconducting Machinery". The CDF coil has a measured RRR = 1885. The curve for RRR = 1000 was taken from CDF Coil Design Note #69. It was calculated by Bob Kephart during the shop test of the CDF coil in Japan; the results appear in his logbook without details--the calculation seems to be in error. The curve for copper, RRR = 50, taken from Wilson is included for comparison.

The details of my calculations are given in the appendix.

COMPARISON TO MEASURED $U(\theta_m)$

The "MIITS" curve was measured for a 10-inch length of CDF conductor and reported in CDF Coil Design Note #69. The MIITS data were converted to $U(\theta_m)$ data by dividing the measured $\int I^2(T)\, dT$ by the area of the aluminum component of the matrix $[(21/23)(3.89 \times 20\ \mathrm{mm}^2)]$. When plotted on Fig. 1 the measured points do not seem consistent with the all-aluminum calculation. They lie close to the copper, RRR = 50, curve. The slope of the measured curve at 250 K is approximately that of the copper curve. Obviously, this suggests that the all-aluminum assumption may not be justified for the CDF conductor. Of course, there may be a systemic error in the measurements--the experiment is not trivial.

CALCULATE $U(\theta_m)$ FOR COPPER-ALUMINUM MATRIX

In his Equation 9.4, Wilson defines the quantities γ, C, and ρ to be "averaged over the winding cross-section". To understand what this means, I looked at the two terms in more detail.

<u>Heat generation term</u>

For a three-component conductor, the LHS of Eq. 2 is

$$\text{Power} = J_{Al}^2 \rho_{Al} V_{Al} + J_{Cu}^2 \rho_{Cu} V_{Cu} + J_{SC}^2 \rho_{SC} V_{SC} \tag{5}$$

$$\text{Power/conductor volume} = \frac{V_{Al}}{V_T} J_{Al}^2 \rho_{Al} + \frac{V_{Cu}}{V_T} J_{Cu}^2 \rho_{Cu} + \frac{V_{SC}}{V_T} J_{SC}^2 \rho_{SC} \tag{6}$$

For the CDF conductor

$$\text{Power/cond vol} = \frac{21}{23} J_{Al}^2 \rho_{Al} + \frac{1}{23} J_{Cu}^2 \rho_{Cu} + \frac{1}{23} J_{SC}^2 \rho_{SC} \tag{7}$$

Noting that $J^2\rho = I^2\rho/A^2 = v^2\rho/(\rho l)^2 = v^2/\rho l^2$, where v is the terminal voltage, Eq. (7) becomes

$$\text{Power/cond vol} = \frac{21}{23}\frac{v_{Al}^2}{\rho_{Al}\,\ell_{Al}^2} + \frac{1}{23}\frac{v_{Cu}^2}{\rho_{Cu}\,\ell_{Cu}^2} + \frac{1}{23}\frac{v_{sc}^2}{\rho_{sc}\,\ell_{sc}^2} \tag{8}$$

Since $l_{Al} = l_{Cu} = l_{sc}$ and $v_{Al} = v_{Cu} = v_{sc}$

$$\text{Power/ cond vol} = \frac{v^2}{\ell^2}\left[\frac{21}{23}\frac{1}{\rho_{Al}(\theta)} + \frac{1}{23}\frac{1}{\rho_{Cu}(\theta)} + \frac{1}{23}\frac{1}{\rho_{sc}(\theta)}\right] \tag{9}$$

Since $\rho_{Al}(\theta) \sim \rho_{Cu}(\theta) << \rho_{sc}(\theta)$, the superconducting contribution to the heating can be ignored. The assumption that

$$(21/23)(1/\rho_{Al}) + (1/23)(1/\rho_{Cu}) = 1/\rho_{Al} \tag{10}$$

is good to about 10%.

The energy generated per unit volume of conductor is approximately $J^2{}_{Al}\ \rho_{Al}(\theta)\ dT$.

This discussion shows that Wilson's "average" is an average weighted by the ratio of the cross sectional areas. The large aluminum to copper ratio justifies my using only the heat generated in the aluminum for the analysis of the CDF conductor.

<u>Heat absorption term</u>

The RHS of Eq. 2 for a three-component matrix is

$$\text{Power} = m_{Al}\,c_{Al}\,d\theta + m_{Cu}\,c_{Cu}\,d\theta + m_{sc}\,c_{sc}\,d\theta$$

$$= \left(V_{Al}\,\gamma_{Al}\,c_{Al} + V_{Cu}\,\gamma_{Cu}\,c_{Cu} + V_{sc}\,\gamma_{sc}\,c_{sc}\right)d\theta \tag{11}$$

$$\text{Power/cond vol} = \left(\frac{V_{Al}}{V_T}\gamma_{Al}\,c_{Al} + \frac{V_{Cu}}{V_T}\gamma_{Cu}\,c_{Cu} + \frac{V_{sc}}{V_T}\gamma_{sc}\,c_{sc}\right)d\theta \tag{12}$$

For the CDF conductor and assuming that $\gamma_{sc}\ C_{sc} = \gamma_{Cu}\ C_{Cu}$,

$$\text{Power/cond vol} = [(21/23)\ \gamma_{Al}\ C_{Al} + (2/23)\ \gamma_{Cu}\ C_{Cu}]\ d\theta \tag{13}$$

<u>Energy balance equation for CDF conductor</u>

The energy generated and absorbed per unit volume for the CDF conductor is

$$J^2{}_{Al}(T)\ \rho_{Al}(\theta)\ dT = [(21/23)\ \gamma_{Al}\ C_{Al}(\theta) + (2/23)\ \gamma_{Cu}\ C_{Cu}(\theta)]\ d\theta \tag{14}$$

and

$$U(\theta_m) = \int_0^\infty J_{Al}^2(T)\,dT = \int_{10K}^{\theta_m} \left(\frac{\frac{21}{23}\gamma_{Al} C_{Al}(\theta) + \frac{2}{23}\gamma_{Cu} C_{Cu}(\theta)}{\rho_{Al}(\theta)} \right) d\theta \tag{15}$$

Calculation of $U(\theta_m)$ for CDF conductor

I calculated $U(\theta_m)$ for the CDF conductor using Eq. 15 and RRR = 1885 aluminum; the details are in the appendix. The results are plotted in Fig. 2. The calculation shows that the copper does have a slight effect on $U(\theta_m)$, about 10 K at 100 K. It is clear that the apparent discrepancy between the measured and calculated $U(\theta_m)$ is not alleviated by including the heat capacity of the copper component of the matrix.

ESTIMATING θ_m FOR A CDF QUENCH

First guess

For a first guess I would estimate the LHS of Eq. 2 as $\int J^2(T) = J_o^2 L/2R_D$ (Wilson's Equation 9.65, pg 219) where R_D is the fast dump resistance, which is assumed to be much larger than the resistance of the normal zone. For I_o = 4500 A and with all the current in the aluminum, J_o = 4500 A/(21/23)(3.89 x 20 mm^2) = 63.34 MA/m^2 = 6334 A/cm^2. The time constant, using the calculated/expected inductance, L/R_D = 2.4 H/0.076 Ω = 31.5 s. Therefore,
$U(\theta_m) = \int J_o^2(T)\,dT = (1/2)(63.34 \times 10^6)^2(31.5) = 7.80 \times 10^{16}\ A^2\text{-s-m}^{-4}$.

At this point in the design the conductor area ratio would be known, but not the aluminum RRR. My first guess therefore would be to get θ_m from $U(\theta_m)$ for an all-aluminum, RRR = 1500 conductor:

θ_m = 110 K for I_o = 4500 A and 195 K for 5000 A.

Second guess

When a piece of conductor became available, the MIITS curve was measured. Using $\int J_o^2(T)\,dT$ from the first guess and the measured $U(\theta_m)$,

θ_m = 107 K for I_o = 4500 A and 150 K for 5000 A.

The large difference between the measured and calculated $U(\theta_m)$ is bothersome because it makes a 30% difference in θ_m at 5000 A.

Third guess

When the completed coil was cooled down to 4.5 K for the first time, the resistance ratio of the coil at 10 K was measured to be 1885. A calculation found in the appendix shows that if the RRR of the copper is between 100 and 250, an aluminum RRR = 2015 explains this measurement. The $U(\theta_m)$ for the composite conductor with aluminum RRR = 2000 would have been calculated and applying the $\int J^2(T)\,dT$ from the first guess θ_m deduced. In my calculation of $U(\theta_m)$ for the composite I used $\rho(\theta)$ for aluminum RRR = 1885. My third guesses, θ_m = 86 K for 4500 A and 145 K for 5000 A, are therefore slight overestimates.

Final guess

During commissioning, the CDF coil was fast dumped from currents ranging from 1 to 5 kA. These tests were described in the paper at CEC-87, "Advances in Cryogenic Engineering", Vol. 31, p.187. The effective time constant for dumps from 1, 1.5, and 2 kA was 30.9 s. This led us to conclude that the coil was superconducting throughout these dumps. I furthermore think that this observed time constant is a good way to calculate the actual inductance: L = (30.9 s)(0.076 Ω) = 2.35 H. (I did not search the test records for the inductance measured during a constant-voltage charge.)

The effective time constant observed during fast dumps from various currents is given in Table 1. The current at which the quench actually started was less that the current at the beginning of the fast dump. For example, during a fast dump from 5 kA the current decayed to 4.6 kA before a quench began. The θ_m given in Table 1 for a 5-kA dump are based on an initial quench current of 4.5 kA, all of which is in the aluminum portion of the conductor. The θ_m for dump currents between 2.9 and 4.5 kA were based on initial quench currents taken from a linear interpolation between the points 2 kA/0 and 5.0/4.5 (figure in the appendix).

NOTES AND CONCLUSION

1. The CDF magnet has no temperature sensors on the coil so it is not possible to measure θ_m directly.

2. From Table 2 one can see that the heat balance method gives a factor-of-two over-estimate at the design stage.

3. I do not have a real explanation for the difference between the measured and calculated $U(\theta_m)$.

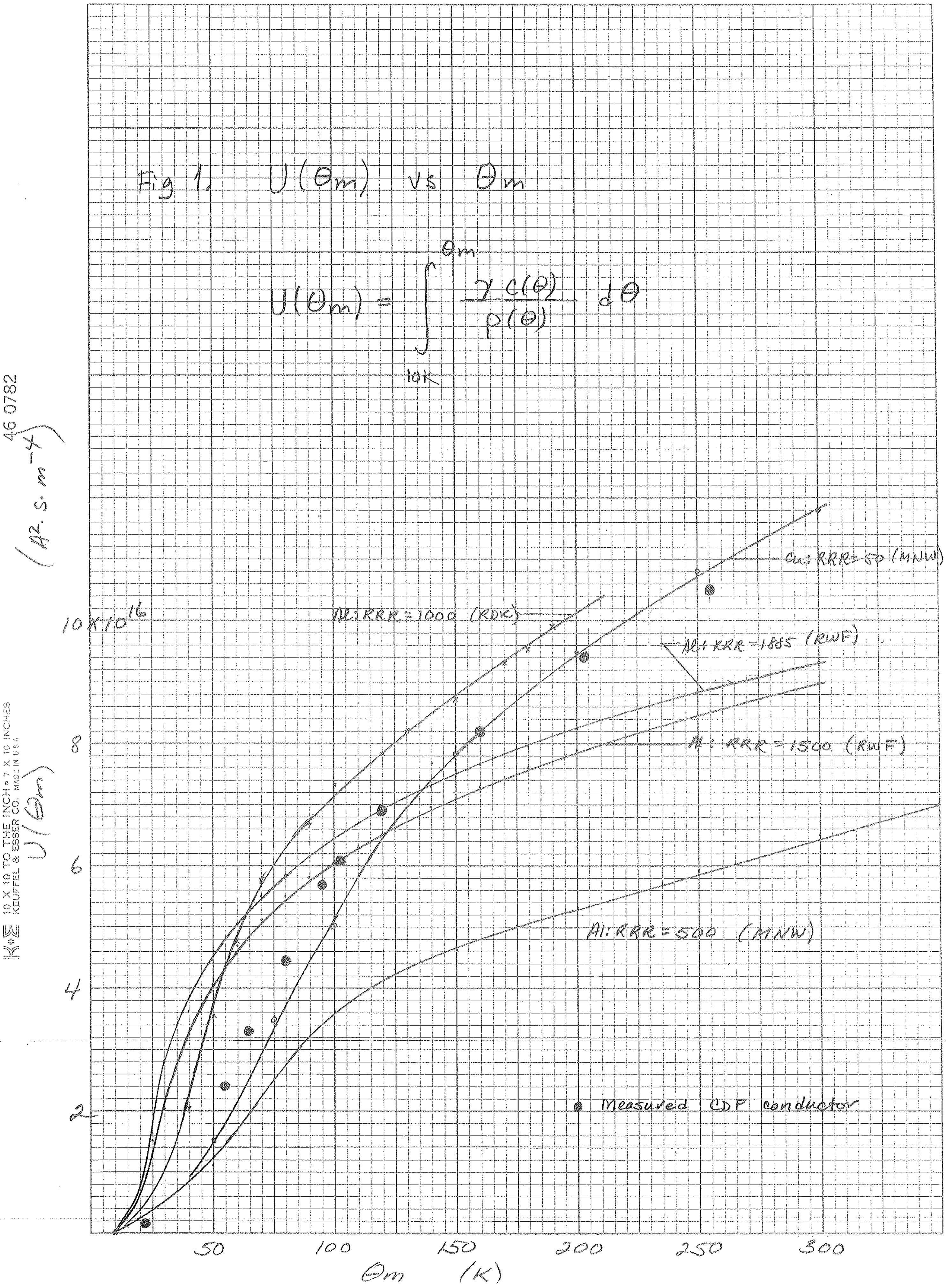
Fig 1. U(θm) vs θm
$U(\theta_m) = \int_{10K}^{\theta_m} \frac{\gamma\, c(\theta)}{\rho(\theta)}\, d\theta$
$(A^2 \cdot s \cdot m^{-4})$
$U(\theta_m)$
10×10^{16}
8
6
4
2
50
100
150
200
250
300
θm (K)
Cu: RRR = 50 (MNW)
Al: RRR = 1000 (RDK)
Al: RRR = 1885 (RWF)
Al: RRR = 1500 (RWF)
Al: RRR = 500 (MNW)
Measured CDF conductor

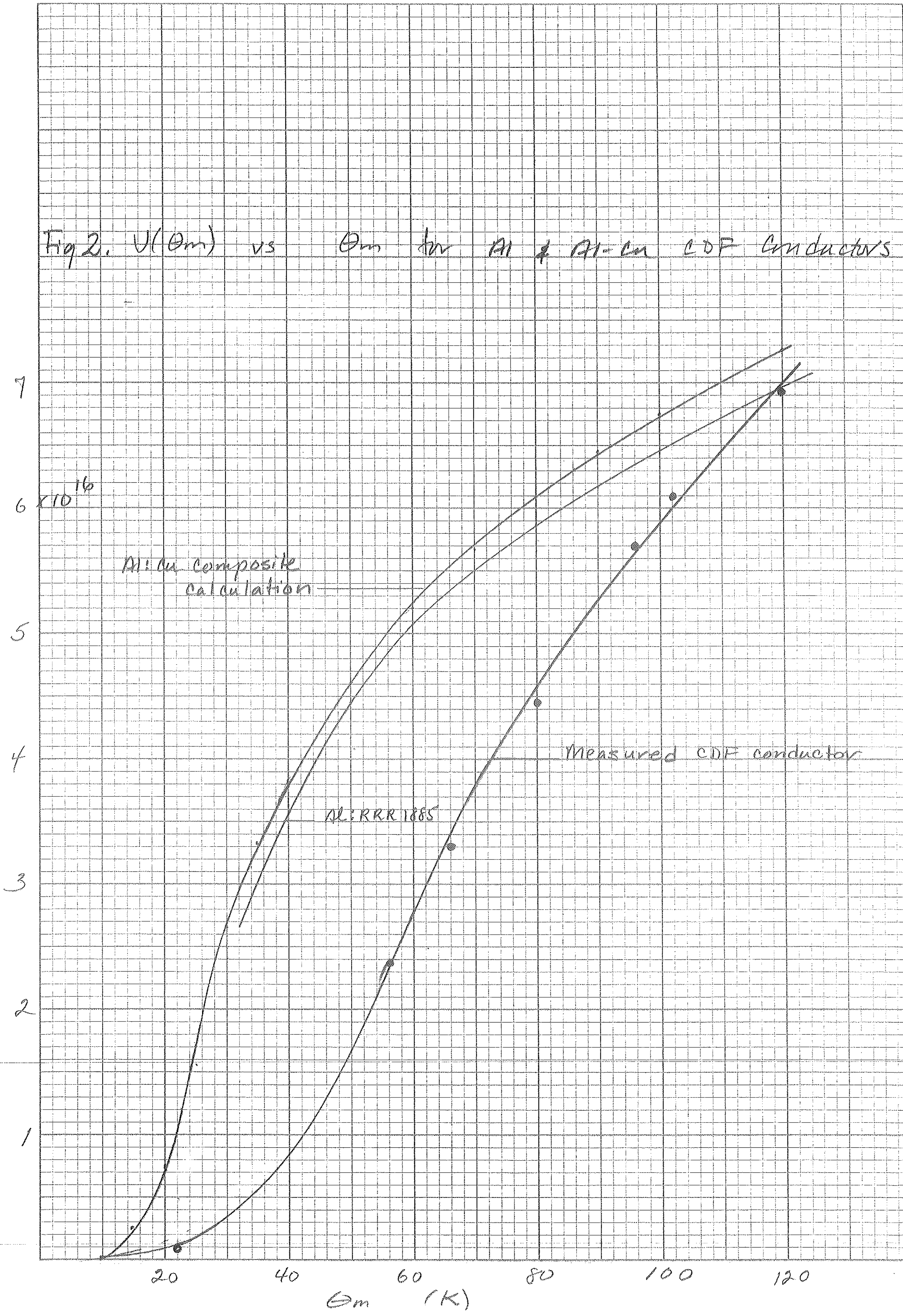

Fig 2. $U(\Theta m)$ vs Θm for Al & Al-Cu CDF Conductors

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ENGINEERING NOTE

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SUBJECT	NAME	
Table 1. Hot spot temp vs fast dump current for CDF coil	RWFast	
	DATE 4/5/89	REVISION DATE

Fast dump initial current	Observed τ_{eff} (1)	Initial quench current	$U(\Theta m)$ (2)	Θm From composite calculation	Θm From measurement
A	s	A	$A^2 \cdot s \cdot m^{-4}$	K	K
			$\times 10^{16}$		
1000	30.9	0	0	<10	<10
1500	30.9	0	0	<10	<10
2000	30.9	0	0	<10	<10
2900	29.4	1300 (3)	0.49	18	34
3500	28.3	2300 (3)	1.48	24	48
4100	27.0	3150 (3)	2.66	30	59
4500	26.0	3800 (3)	3.72	39	69
5000	19.5	4500	3.91	42	71

(1) From "Advances in Cryogenic Engineering" Vol 31, p. 187

(2) $U(\Theta m) = \frac{1}{2} J_0^2 \tau_{eff} = \frac{1}{2} \frac{I_{0Q}^2 \tau_{eff}}{A_{Al}^2} = \frac{I_{0Q}^2 \tau_{eff}}{1.009 \times 10^{-8}}$

(3) From linear extrapolation - see Appendix D.

FERMILAB ENGINEERING NOTE

SECTION	PROJECT	SERIAL-CATEGORY	PAGE
		DN29	

SUBJECT: Table 2. Hot spot temp for CDF conductor / coil

NAME: RW FAST

DATE: 2/5/89

REVISION DATE:

$I_0 = 4500A$ & $5000A$

Method, τ, $U(\Theta m)$	$U(\Theta m)$ (K) Alum RRR 1500	Alum RRR 1865	Al-Cu composite	Measured
Design & Conductor testing $I_{0Q} = 4500A$ $\tau = L_{calc}/R_D = 31.5$ s $U(\Theta m) = 6.32 \times 10^{16}$	110	95	86	107
Magnet testing $I_{0Q} = 4500A$ $\tau = 19.5$ (measured) $U(\Theta m) = 3.91 \times 10^{16}$	47	43	41	71
$I_{0Q} = 5000A$ $\tau = 31.5$ s $U(\Theta m) = 7.6 \times 10^{16}$	195	170	145	150

Appendix A. $U(\theta_m)$ for all-aluminum conductor

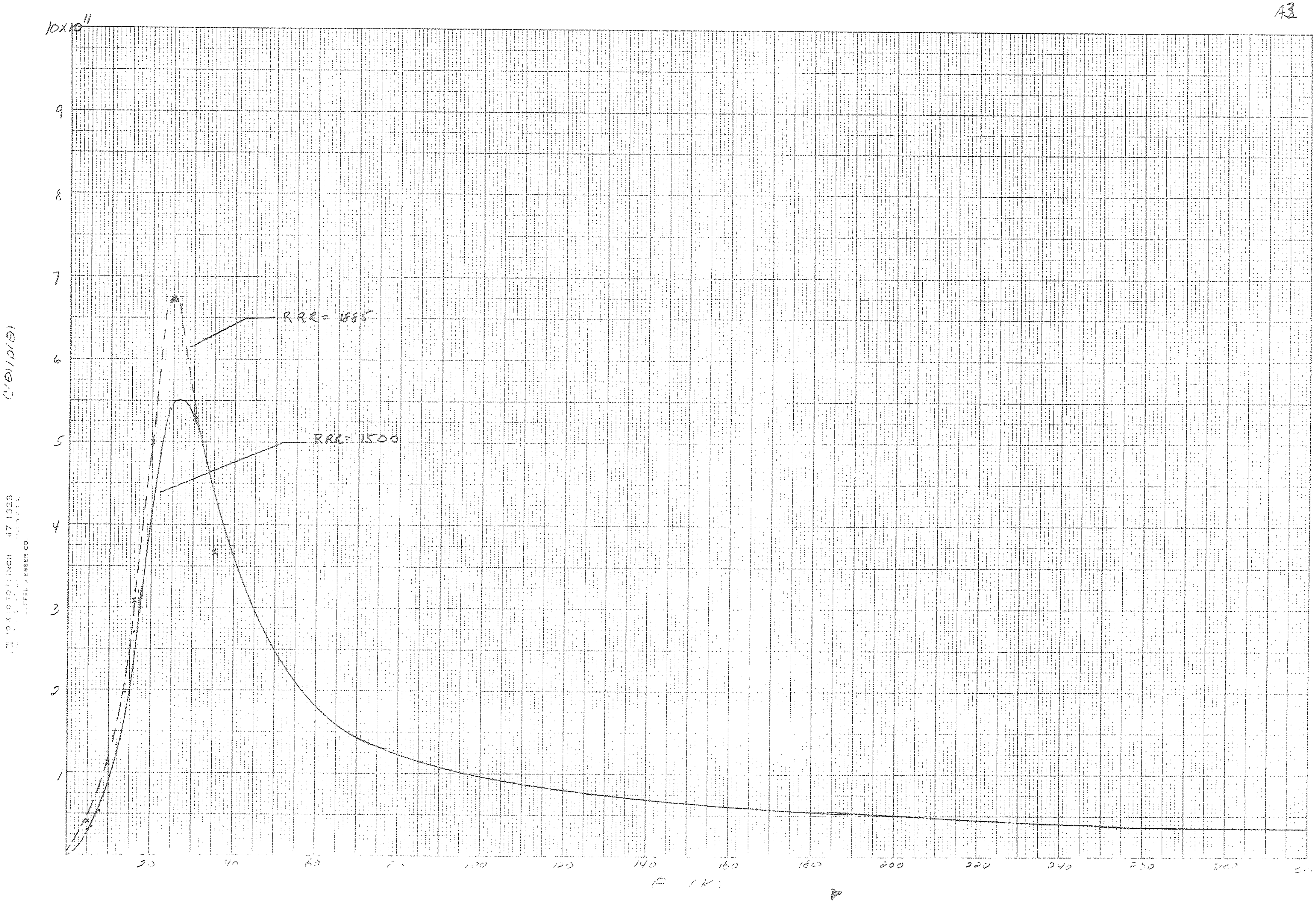
Temp θ (K)	$c(\theta)$ J/kg-K	$\rho(\theta)$ Ω-m RRR 1885 $\times 10^{-11}$	1500	$\frac{c(\theta)}{\rho(\theta)}$ 1885 $\times 10^{11}$	1500
5	0.5	1.28	1.62	.39	.31
6	0.6	↓	↓	.47	.37
8	0.9	↓	↓	.70	.56
10	1.4	↓	↓	1.09	.86
12	2.15	1.3	↓	1.65	1.33
14	3.2	1.4	1.65	2.29	1.94
16	4.6	1.5	1.70	3.07	2.71
18	6.3	1.6	1.85	3.94	3.40
20	8.5	1.7	2.1	5.00	4.05
25	17.5	2.6	3.2	6.73	5.47
30	32	6	6	5.33	5.33
35	55	15	15	3.67	3.67
40	80	22	22	3.64	3.64
50	150	51	51	2.94	2.94
60	210	130	130	1.62	1.62
70	290	200	200	1.45	1.45
80	365	300	300	1.22	1.22
90	420	400	400	1.05	1.05
100	480	463	463	1.04	1.04
150	660	1000	1000	.68	.68
200	780	1600	1600	.49	.49
250	830	2200	2200	.37	.37
300	860	2430	2430	.35	.35

A2

All-aluminum conductor

θ_m (K)	$\int_{10K}^{\theta_m} \frac{C(\theta)}{\rho(\theta)}\, d\theta$ RRR=1885	RRR=1500	$U(\theta_m) = \gamma \int_{10K}^{\theta_m} \frac{C(\theta)}{\rho(\theta)}\, d\theta$ RRR=1885	RRR=1500
10	0	0	0	0
15	9.25×10^{11}	9.25×10^{11}	0.25×10^{16}	0.25×10^{16}
20	27.75	24.5	0.75	0.66
25	57.75	48.25	1.56	1.30
30	88.75	75.25	2.40	2.03
40	134.45	121.0	3.63	3.27
50	165.25	150.45	4.46	4.06
60	186.75	173.3	5.04	4.68
70	203.00	189.5	5.48	5.12
80	216.4	203.0	5.84	5.48
90	227.8	214.4	6.15	5.79
100	238.0	224.5	6.43	6.06
120	255.6	242.2	6.90	6.54
140	270.4	256.9	7.30	6.94
160	283.2	269.7	7.65	7.28
180	294.8	281.3	7.96	7.60
200	305.2	291.7	8.24	7.88
250	327.0	313.5	8.83	8.46
300	345.0	331.5	9.32	8.95

A3



10x10^11
9
8
7
6
5
4
3
2
1
RRR = 1885
RRR = 1500
20
40
60
100
120
140
160
180
200
220
240

A4

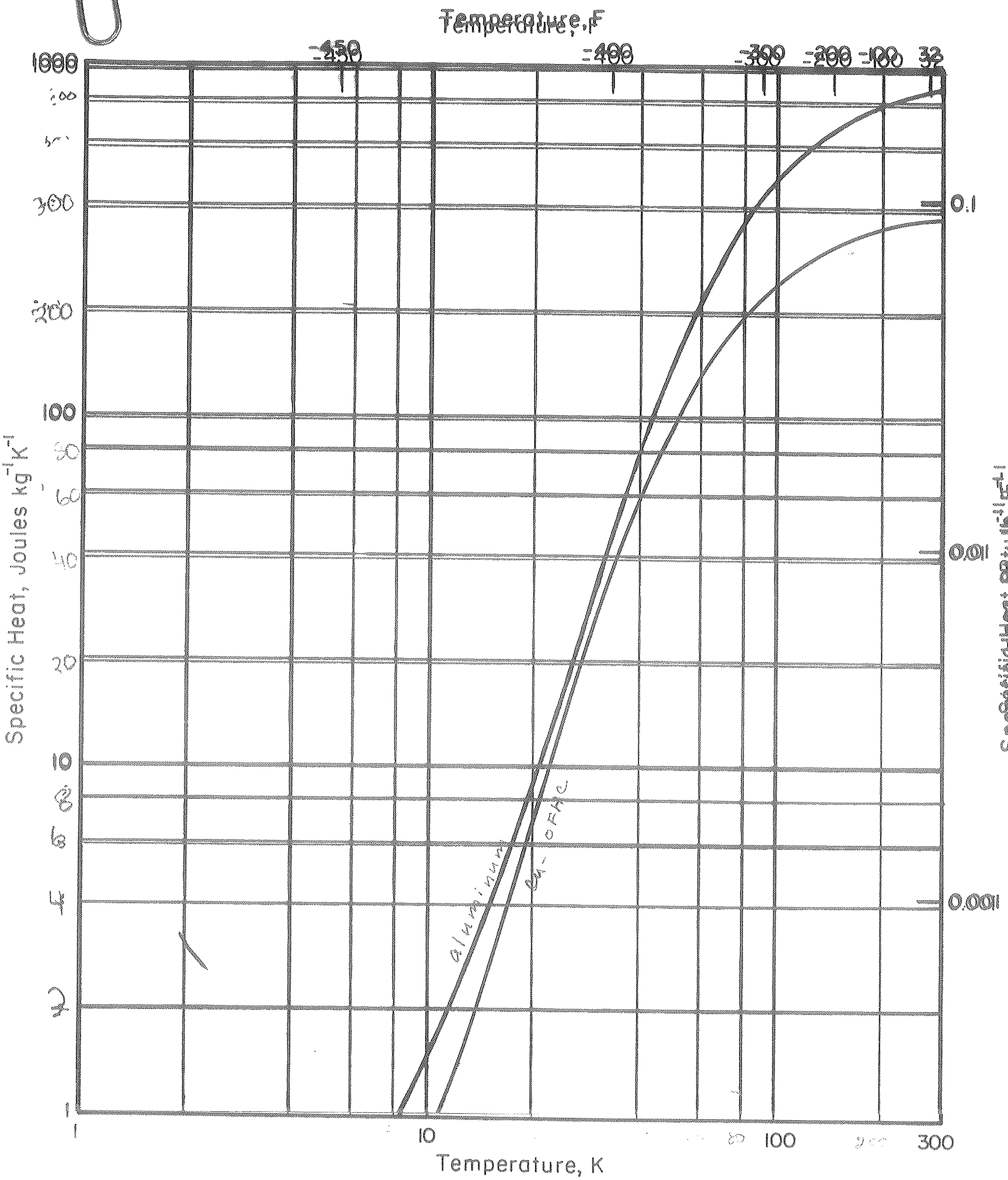
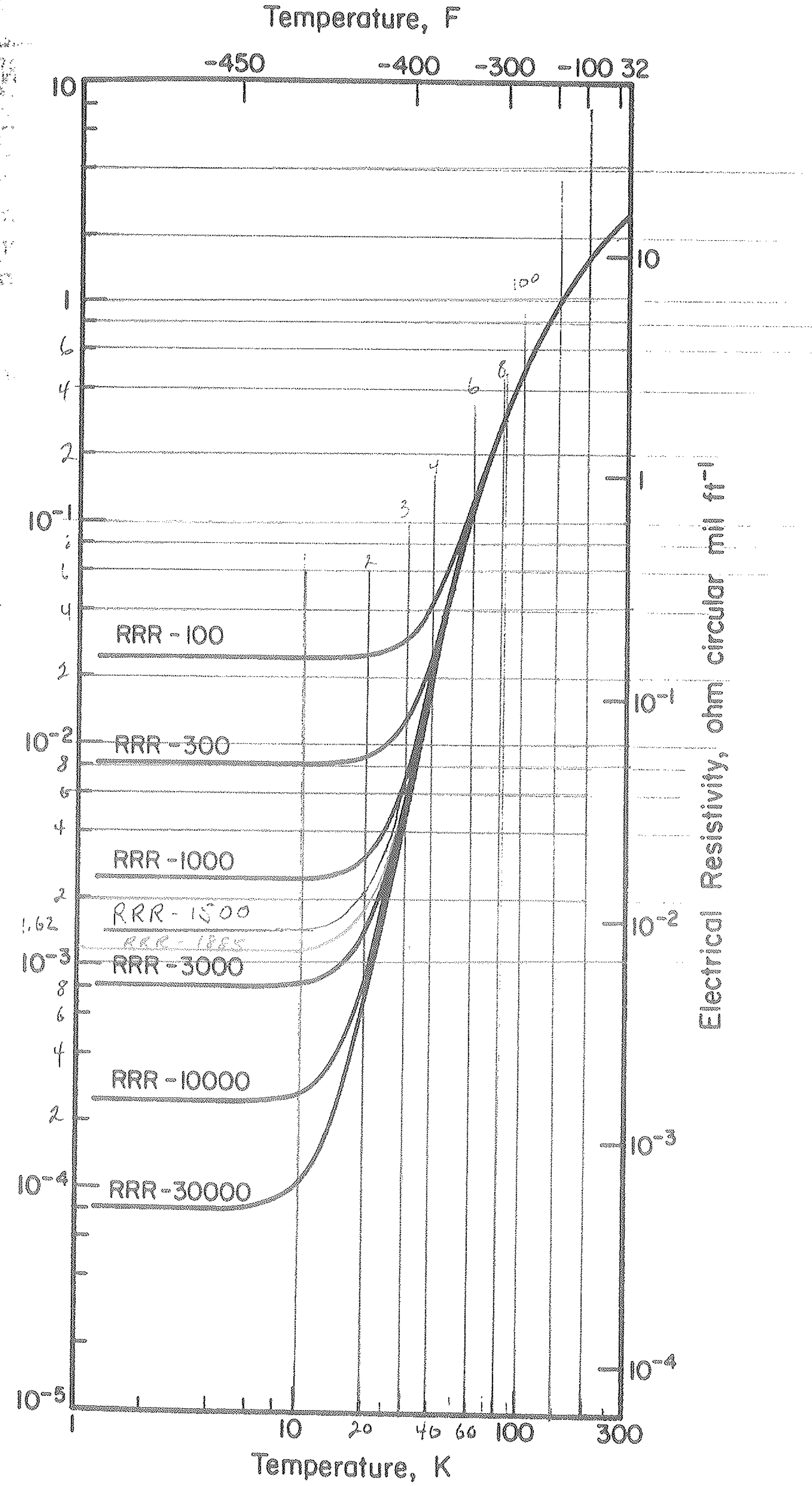


FIGURE 4.1.1-S3. SPECIFIC HEAT VERSUS TEMPERATURE FOR ALUMINUM AND OFHC COPPER

Note: This figure [illegible] 3 x 3 log log [illegible] graph paper

A5

A2



80°

FIGURE 4.1.1-R1. ELECTRICAL RESISTIVITY VERSUS TEMPERATURE FOR ALUMINUM

Wilson's Aluminium curve - Fig 9.1

$\rho_0 = 5 \times 10^{-11}\ \Omega \cdot m$; RRR = 486 ~ 500

Θ_m	$U(\Theta)$
25 K	$0.4 \times 10^6\ A^2 \cdot s \cdot m^{-4}$
50	1.1
75	2.5
100	3.6
125	4.3
150	4.6
175	5.0
200	5.3
225	5.6
250	5.9
300	6.5
350	7.0

Kepharts MIITS Curve

Taken from RDK's Hitachi logbook

θ_{max}	$\int_0^{\theta m} \frac{C(\theta)}{\rho(\theta)}\, d\theta$	$\frac{1}{A_{Al}^2} \int_0^{\theta m} \frac{C(\theta)}{\rho(\theta)}\, d\theta$ (2) $= U(\theta_{max})$
20	10×10^6 (1)	0.20×10^{16} $A^2 \cdot s \cdot m^{-4}$
30	30	.59
40	100	1.98
50	180	3.56
60	240	4.75
70	290	5.74
90	340	6.73
100	370	7.33
120	395	7.82
130	415	8.22
150	440	8.71
170	470	9.31
180	480	9.50
190	500	9.90

(1) The units are not clear because Bob does not give the units on the specific heat or resistivity. He equates $\int \frac{c}{\rho} d\theta$ to $\int I^2 dt$ so I assume to get $U(\theta)$ I must divide by area^2. This is very iffy because it appears that Bob did not multiply by the density; and omitting the density is not the same as omitting the conductor area.

(2) $A_{Al} = 21/23\,(20 \times 3.89\ mm) = 7.10 \times 10^{-5} m^2$; $A_{Al}^2 = 50.5 \times 10^{-10}\ m^4$

Appendix B. MIITS measurement - CDF conductor

Measured $U(\Theta_{max})$: CDF Coil Design Note 69

Θ_{max} (K)	$\int_0^\infty I^2 dT$ ($A^2 \cdot s$)	$\int_0^\infty J^2 dT$ ($A^2 \cdot s \cdot m^{-4}$)
10	0	0
23	38.6×10^6	$.076 \times 10^{16}$
56	120	2.38
66	167	3.31
80	225	4.46
96	288	5.70
102	307	6.08
120	350	6.93
160	414	8.20
202	476	9.43
255	531	10.51

$$\int_0^\infty J^2 dT = \frac{1}{A_{Al}^2} \int_0^\infty I^2 dT$$

$$= \left[\left(\frac{21}{23}\right)\left(20 \times 3.89 \times 10^{-6} m^2\right)\right]^{-2} \int_0^\infty I^2 dt$$

$$= \frac{1}{50.5 \times 10^{-10} m^4} \int_0^\infty I^2 dt$$

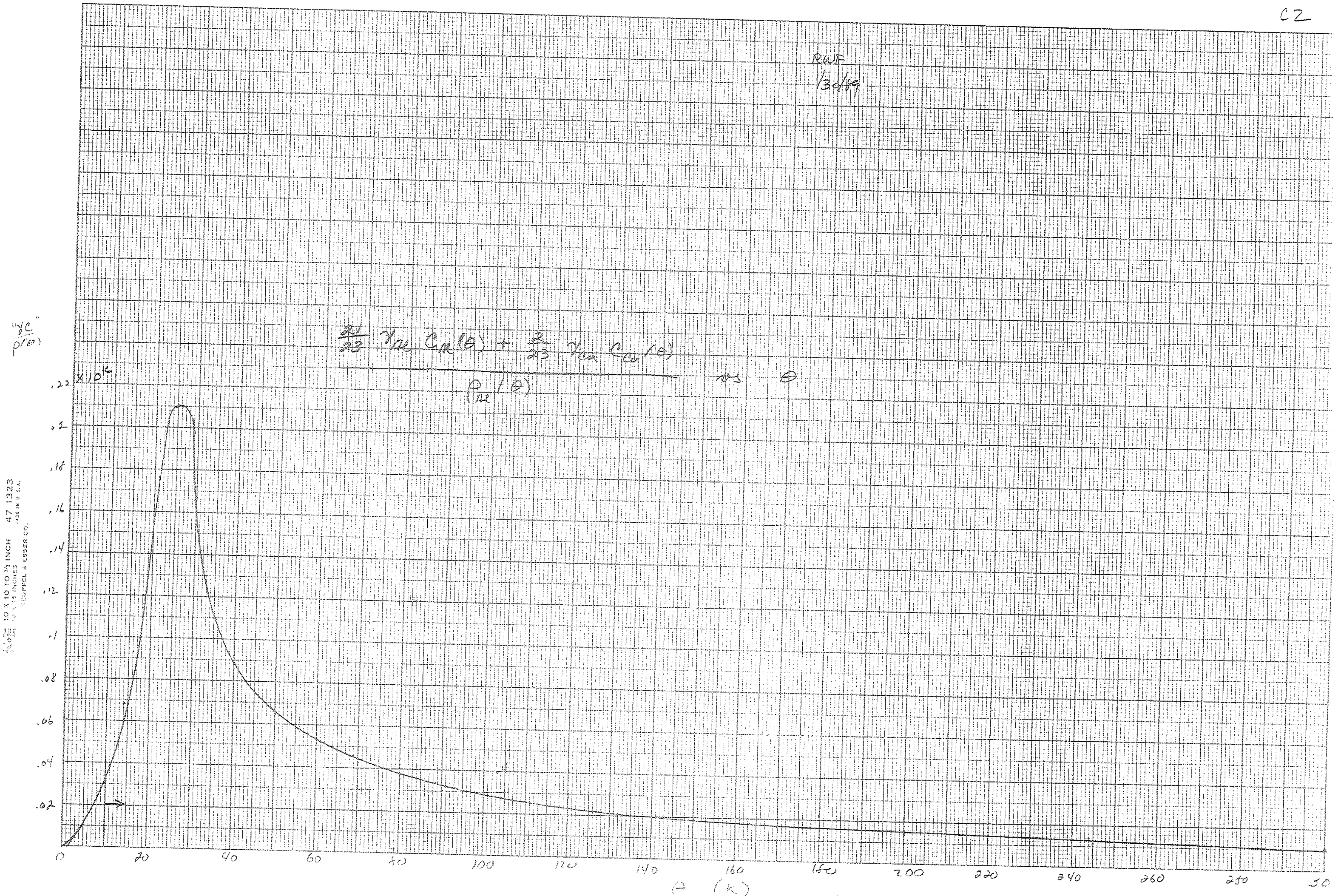
Appendix C. U(Θm) for Al-Cu composite conductor

Temp Θ (K)	Aluminum C J/kg.K	γ kg/m³	$\frac{21}{23}\gamma C$	Copper C	$\frac{2}{23}\gamma C$	"γC"	RRR 1885 "γC"/ρ(Θ)
						$\times 10^{-4}$	
5	.15	$.27\times10^{4}$	$.123\times10^{-4}$	.15	$.012\times10^{-4}$	.135	$.105\times10^{15}$
6	.6	↓	.148	.25	.019	.167	.130
8	.9		.22	.5	.038	.258	.202
10	1.4		.35	.65	.066	.416	.325
12	2.15		.53	1.35	.10	.63	.48
14	3.2		.78	2.2	.17	.95	.68
16	4.6		1.13	3.4	.26	1.39	.93
18	6.3		1.55	5.0	.39	1.94	1.21
20	8.5		2.10	7.0	.54	2.64	1.55
25	17.5		4.31	15	1.16	5.47	2.10
30	32		7.89	27	2.09	9.98	1.66
35	55		13.56	43	3.33	16.9	1.13
40	80		19.7	60	4.64	24.3	1.10
50	150		37.0	95	7.36	44.4	.87
60	210		51.8	130	10.1	61.9	.48
70	290		71.5	170	13.2	84.7	.42
80	365		90.0	200	15.5	105.5	.35
90	420		103.5	220	17.0	120.7	.30
100	480		118.3	250	19.4	137.7	.297
150	680		167.6	320	24.8	192.4	.19
200	780		192.3	360	27.9	220.2	.14
250	830		204.6	375	29.1	233.7	.11
300	860		212.0	380	33.8	245.8	.10

$\gamma_{Al} = 2.7\ g/cm^3 = .27\times10^4\ kg/m^3$; $\gamma_{Cu} = .89\times10^4\ kg/m^3$

$$"\gamma C" = \frac{21}{23}\gamma_{Al} C_{Al} + \frac{2}{23}\gamma_{Cu} C_{Cu}$$

C2

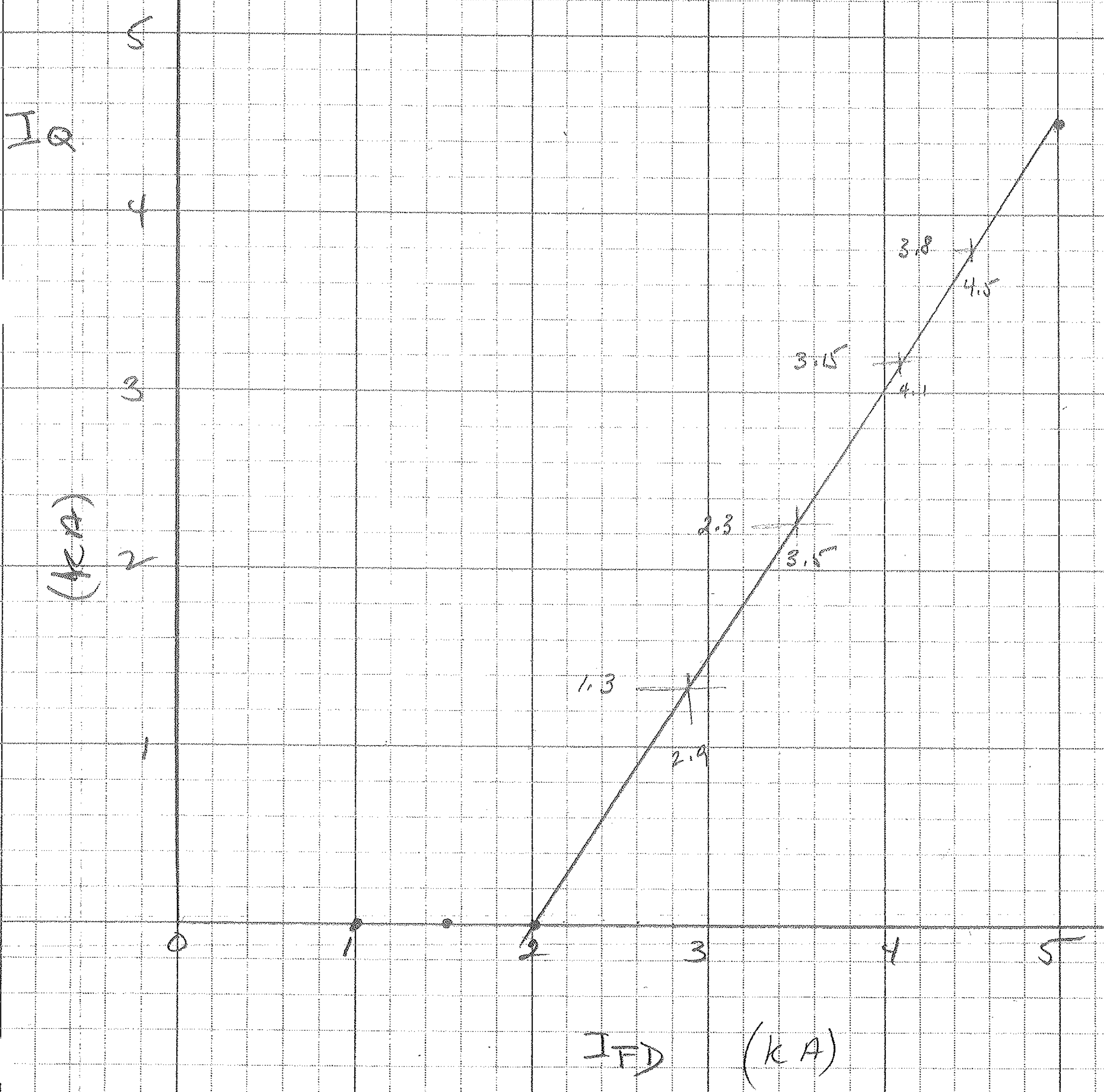

RWF
1/30/89
"γC"
ρ(θ)
x10^6
.22
.2
.18
.16
.14
.12
.1
.08
.06
.04
.02
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
θ (K)

composite CDF conductor

$\frac{21}{23}$ Al RRR = 1885 + $\frac{2}{23}$(Cu + SC)

Θ_m (K)	$U(\Theta_m) = \int_{10K}^{\Theta_m} \left(\frac{21}{23} \gamma_{Al} C_{Al}(\theta) + \frac{2}{23} \gamma_{Cu} C_{Cu}(\theta) \right) / \rho_{Al}(\theta) \, d\theta$ $A^2 \cdot s \cdot m^{-4}$
10	0
15	0.24×10^{16}
20	0.74
25	1.70
30	2.65
35	3.33
40	3.82
50	4.60
60	5.19
70	5.68
80	6.09
90	6.44
100	6.75
120	7.27
140	7.70
160	8.08
180	8.42
200	8.73
250	9.40
300	9.96

Appendix D. Quench current vs fast dump current – CDF coil



Question: If the coil average RRR = 1885, what is the RRR of the aluminum component?

The actual measurement was

$$\frac{R_T(300K)}{R_T(10K)} = 1885$$

For 3 conductor components in parallel

$$\frac{1}{R_T} = \frac{1}{R_{Al}} + \frac{1}{R_{Cu}} + \frac{1}{R_{SC}}$$

$$\boxed{\frac{l}{R_T} = \frac{A_{Al}}{\rho_{Al}} + \frac{A_{Cu}}{\rho_{Cu}} + \frac{A_{SC}}{\rho_{SC}}}$$

$\rho_{SC} >> \rho_{Cu}, \rho_{Al}$ at all temps, so drop SC term

$$\frac{l}{R_T} = \frac{A_{Al}}{\rho_{Al}} + \frac{A_{Cu}}{\rho_{Cu}} = \frac{A_T}{\rho_{avg}}$$

$R = \frac{\rho l}{A}$

$\frac{1}{R} = \frac{A}{\rho l}$

$$\frac{l}{R_T} = \frac{A_{Al}}{\rho_{Al}} + \frac{A_{cu}}{\rho_{cu}}.$$

$$= \frac{A_{Al}\rho_{cu} + A_{cu}\rho_{Al}}{\rho_{Al}\,\rho_{cu}}$$

$$\boxed{\frac{R_T}{l} = \frac{\rho_{Al}\,\rho_{cu}}{A_{Al}\,\rho_{cu} + A_{cu}\,\rho_{Al}}}$$

at 300 K ; $\rho_{Al}(300) = 2.43 \times 10^{-8}$, $\rho_{cu}(300) = 1.55 \times 10^{-8}$ $\Omega \cdot m$

$$\frac{R_T}{l}(300K) = \frac{(2.43 \times 10^{-8})(1.55 \times 10^{-8})\ \Omega^2 \cdot m^2}{(71.03 \times 10^{-6} m^2)(1.55 \times 10^{-8}\ \Omega \cdot m) + (3.38 \times 10^{-6})(2.43 \times 10^{-8}\ \Omega \cdot m)}$$

$$= \frac{3.767 \times 10^{-16+14}}{118.31 \times 10^{-14}}$$

$$= .032 \times 10^{-2}\ \Omega/m = .000320$$

$$= 320\ \mu\text{-}\Omega/m$$

Assume ~~RRR = 215~~

$$\frac{R_T}{l}(10K) = \frac{\rho_{Al}(10)\,(7.3\times10^{-11})}{(71.03\times10^{-6})(7.3\times10^{-11}) + (3.38\times10^{-6})\,\rho_{Al}(10)}$$

$$= \frac{7.3\times10^{-11}\,\rho_{Al}(10)}{5.185\times10^{-15} + 3.38\times10^{-6}\,\rho_{Al}(10)}$$

$$\frac{R_T/l\,(300)}{R_T/l\,(10)} = 1885 = \frac{320\times10^{-6}}{\dfrac{7.3\times10^{-11}\,\rho_{Al}(10)}{5.185\times10^{-15} + 3.38\times10^{-6}\,\rho_{Al}(10)}}$$

$$1885 = \frac{320\times10^{-6}\left(5.185\times10^{-15} + 3.38\times10^{-6}\,\rho_{Al}(10)\right)}{7.3\times10^{-11}\,\rho_{Al}(10)}$$

$$1.376\times10^{-7}\,\rho_{Al}(10) = 1.659\times10^{-18} + 1.082\times10^{-9}\,\rho_{Al}(10)$$

$$\rho_{Al}(10)\left[1.376\times10^{-7} - \cancel{1.082\times10^{-9}}\right] = 1.659\times10^{-18+7}$$

$$\rho_{Al}(10) = 1.206\times10^{-11}\ \Omega\cdot m$$

$$\frac{\rho_{Al}(300K)}{\rho_{Al}(10)} = \frac{2.43\times10^{-8}}{1.206\times10^{-11}} = 2.015\times10^{3}$$

$$\boxed{= 2015}$$

For Cu RRR = 100, $\rho_{cu}(10K) = 1.55 \times 10^{-10}\ \Omega \cdot m$

And

$$\frac{R_T/l\,(300)}{R_T/l\,(10)} = 1885 = \frac{320 \times 10^{-6}}{\dfrac{1.55 \times 10^{-10}\ \rho_{Al}(10)}{1.1 \times 10^{-14} + 3.38 \times 10^{-6}\rho_{Al}(10)}}$$

$$2.922 \times 10^{-7}\rho_{Al}(10) = 3.52 \times 10^{-18} + 1.082 \times 10^{-9}\rho_{Al}(10)$$

$$\rho_{Al}(10) = \frac{3.52 \times 10^{-18}}{2.922 \times 10^{-7}} = 1.205 \times 10^{-11}\ \Omega \cdot m$$

$$\frac{\rho_{Al}(300K)}{\rho_{Al}(10K)} = \frac{2.43 \times 10^{-8}}{1.205 \times 10^{-11}} \doteq 2017$$

So Al RRR needed to get 1885 for the conductor does not depend on Cu RRR much

This calculation assumes ρ_{Al} & ρ_{cu} are constant over length.