SSC DETECTOR SOLENOID DESIGN NOTE #4

TITLE: Simple Calculation of Ampere-turns and Stored Energy

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I. Ampere-turns required for idealized solenoid

A. General

The field, H_o, at the axial center of a <u>continuous</u>, <u>infinitesmally thin</u> solenoid of length ℓ and radius b, is

$$H = I n' \ell / (\ell^2 + 4b^2)^{1/2},$$

where

I = conductor current

n' = axial turns density.

$$B = \mu_o H$$
$$= \mu_o I n' \ell / (\ell^2 + 4b^2)^{1/2}$$
$$= \mu_o N I / (\ell^2 + 4b^2)^{1/2},$$

where

N = total number of turns = $n'\ell$

B. 1987 Berkeley Design

Applying the above equation to the entire, 16-m coil, which is neither continuous nor infinitesmally thin (b = avg. radius),

$$B = \frac{(4\pi \times 10^{-7})(8 \times 642 \text{ turns})(5000 \text{ A})}{\left[(16 \text{ m})^2 + 4(4.35 \text{ m})^2\right]^{1/2}}$$

$$= 1.77 \text{ T}$$

This is probably about correct, since the iron should add about 10% to the field.

Note that if you apply the equation to a single module, the result is only ~ 0.45 T; so one must consider the magnet as a unit, at least in these simple calculations.

C. Variation of NI with diameter

$$B = \mu_0 NI / (\ell^2 + 4b^2)^{1/2}$$

$$NI = \mu_0^{-1} B (\ell^2 + 4b^2)^{1/2}$$

For $B = 1.77$ T, $\ell = 16$ m and b = average radius

$$NI = 1.41 \times 10^6 (256 + 4b^2)^{1/2}$$

b	NI (10^6 A-t)	
4.0	19.3	
4.35	25.6	1987 Berkeley design
4.5	25.9	
4.75	26.2	
5.0	26.6	
5.25	27.0	

II. Stored energy

A. In general, $E = \frac{1}{2} \int_{\text{space}} \vec{B} \cdot \vec{H} \, d\text{volume}$

$$= \frac{1}{2\mu_0 \mu} \int_{\text{space}} B^2 \, d\text{volume}$$

B. 1987 Berkeley design

Assume $B = 2\,T$ over entire aperture ($8.2\,m\ \phi \times 16\,m$)

and $B = 1.5\,T$ in the flux return iron

$$E = \frac{1}{2\mu_0} \left[(2T)^2 \left(\tfrac{1}{4}\pi\, 8.2^2\right)(16) \right]$$

$$+ \frac{1}{5000\mu_0} \int_{\text{iron}} (1.5)^2 \, d\text{volume-iron}$$

$$= 1.34 \times 10^9 + \text{Negligible } J$$

The 1987 Berkeley report quotes $E = 1\,GJ$, so the assumption of $B = 2\,T$ over entire aperture is probably an overestimate.

C. For $\phi = 10\,m$

$$E = (1)\left(\frac{10}{8.2}\right)^2 GJ = 1.5\,GJ$$