SSC DETECTOR SOLENOID DESIGN NOTE #16

TITLE: Mutual Inductances of Adjacent 2, 4, and 8-m long, 8.2-m Diameter Solenoids

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DATE: Mar. 29, 1988

I. Reference: Smythe, page 348, problem #21, the mutual inductance of two coaxial solenoids in air:

$$M = \frac{1}{6}\mu_0 n m d^3 \sum_{n=1}^{4} (-1)^n \sec\alpha_n \left[(1-\tan^2\alpha_n) E(k_n) + \tan^2\alpha_n K(k_n) \right]$$

where d = diameter

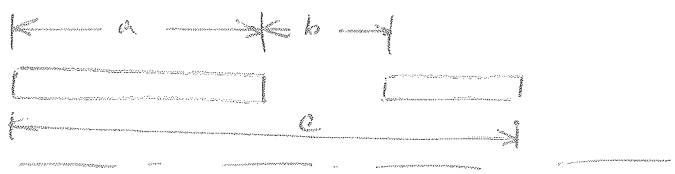
n, m = turns / length

$\tan\alpha_1 = c/d$

$\tan\alpha_2 = (c-a)/d$

$\tan\alpha_3 = b/d$

$\tan\alpha_4 = (a+b)/d$



Caveat: This equation gives negative values for the mutual inductance, the absolute value seems OK.

<u>For the case</u>

$d = 8.2\,m$, $b = 0$, (no coil-to-coil separation)

$a = c - a - b$ (coils of equal length)

Length, a (m)	c (m)	M
2	4	2.2 H
4	8	4.5
8	16	7.15

<u>For the case</u>

$d = 10, \quad a = 8, \quad c = 2a + b$

$b = 0, 0.25, 0.5, 1.0, 2, 4, 8, 12, 16$ m

b (m)	M (H)
0	12.5
0.25	11.5
0.50	10.7
1.0	9.2
2.0	7.1
4.0	4.7
8.0	< 0.1
12.0	< 0.1
16.0	< 0.1

So for adjacent coils, we can neglect the effect of the coil-to-coil separation.

II. Mutual inductances calculated from the self inductances

Because of the "sign problem" with the results using Smythe's equation for M, I recalculated them using

$$L_T = L_1 + L_2 + 2M_{12}$$

In the case of no separation, this must be true:

$$L(16\,m) = L(2 \times 8\,m)$$
$$L(8\,m) = L(2 \times 4\,m)$$
$$L(4\,m) = L(2 \times 2\,m)$$

For example:

$$L(16) = L(8) + L(8) + 2\,M(8,8) = 89.1\ H$$

(self inductances from DN #15)

and $M(8,8) = 7.25\ H$

Results:

Coil length (m)	M between identical, adjacent coils with no separation (H)
2	2.2
4	4.45
8	7.25

which is in very good agreement with the calculation given in I.

Note: (1) These mutual inductances are <u>only</u> for two, identical, adjacent coils with no coil-to-coil separation.

(2) These are NO-IRON calculations.

III. Computer calculation of M

With $E(k_n) = \frac{\pi}{2}\left[1 - \left(\frac{1}{2}\right)^2 k_n^2 - \left(\frac{3}{2\cdot 4}\right)^2 \frac{k_n^4}{3} - \left(\frac{3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \frac{k_n^6}{5} - \cdots\cdots\right]$

And $K(k_n) = \frac{\pi}{2}\left[1 + \left(\frac{1}{2}\right)^2 k_n^2 + \left(\frac{3}{2\cdot 4}\right)^2 k_n^4 + \left(\frac{3\cdot 5}{2\cdot 4\cdot 6}\right)^2 k_n^6 + \cdots\right]$

Bob Wand's wrote a computer program (INDUCT) to calculate Ms.

The results for 2, 4 & 8-m modules are given below. In all these cases there is no separation between immediately adjacent coil modules.

Note about program: If one enters $c = 0$ the program computes the self inductance of the coil of length a. (I haven't been able so far to prove this from Smythe's equations.)

MUTUAL INDUCTANCE

n = m = 381 turns/m
d = 0.2 m
separation between immediately adjacent coils = 0

2-m coil modules

	1	2	3	4	5	6	7	8
1	4.8	2.3	0.88	0.46	0.26	0.16	0.10	0.07
2	2.3	4.8	2.3	0.88	0.46	0.26	.016	0.10
3	0.88	2.3	4.8	2.3	0.88	0.46	0.26	0.16
4	0.46	0.88	2.3	4.8	2.3	0.88	0.46	0.26
5	0.26	0.46	0.88	2.3	4.8	2.3	0.88	0.46
6	0.16	0.26	0.46	0.88	2.3	4.8	2.3	0.88
7	0.10	0.16	0.26	0.46	0.88	2.3	4.8	2.3
8	0.07	0.10	0.16	0.26	0.46	0.88	2.3	4.8

4-m coil modules

1	14.2	4.5	1.13	0.43				
2	4.5	14.2	4.5	1.13				
3	1.13	4.5	14.2	4.5				
4	0.43	1.13	4.5	14.2				

8-m coil modules

1	37.3	7.2						
2	7.2	37.3						

16-m coil

1	87.1							