Mechanical Analysis of Tohoku Bubble Chamber Magnet Helium Dewar Vent Piping

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Introduction

The Tohoku Bubble Chamber magnet uses two liquid helium dewars. Each dewar is protected from over pressure by a 4" rupture disk, a 1" check valve and a 1-1/4" check valve set at 20, 5 and 8 psi, respectively. The valves exhaust into 6" sched 5 vent pipes which lead outside (see Fig. 1). The purpose of this note is to determine the forces, displacements and stresses in both vents due to the inertia of the exhausting gas, the internal pressure, and thermal contraction during venting.

Outline of Solution

Designating the separate vents as vent A and vent B (see Fig. 1),

I. Divide each vent run into components for which flow resistances can be calculated.

II. Calculate mass flow rate and total pressure drop of each vent.

Assumptions:
- a) Dewar is 100 psig reservoir of 10 K gas. (worse case scenario).
- b) Venting is through 4" rupture disk only.
- c) Flow is compressible and isothermal.
- d) Charts for specific heat ratio k = 1.4 are applicable (Actual k = 2.33 at 7 atm and 10 K).
- e) Resistance of flanges is negligible.

III. Calculate component pressure drops by proportioning the total pressure drop according to component resistance.

IV. Calculate the entrance and exit velocities for each piping component from continuity and assumed 10 K uniform temperature.

V. Calculate the forces on the piping components from momentum, using the previously calculated pressures and velocities.

VI. Create a finite element model and apply the calculated inertial and pressure forces. Calculate thermal contraction with the model by assuming the entire vent will cool from 293 K to 10 K during venting.

The following material properties are used for the Type 304 stainless steel vent piping:[1]

1. Yield strength = 30,000 psi.

2. Young's modulus = $29(10^6)$ psi.

3. Thermal contraction from 300 K to 10 K = 0.003 in/in.

I. Vent Pipe Components and Resistances

The vent lines were divided into components for which a flow resistance K could be calculated[2]. Table I gives the dimensions, and resistances of the components illustrated in Fig. 2.

II. Calculation of Mass Flow Rate and Total Pressure Drop

This calculation was done using the assumptions and procedures of Example 4-21 of Ref. 2. The calculation for vent B only will be presented in detail.

The equation for flow is the Darcy formula with expansion factor:

$$W = 0.525\, Yd^2 \left(\frac{\Delta P}{K\bar{V}_1}\right)^{1/2}$$

where
- W = flow rate, lbs/sec
- Y = expansion factor
- d = diameter of vent = 6.407 in
- ΔP = pressure drop across vent
- K = resistance coefficient of vent = 8.0 (vent B)
- $\bar{V}_1$ = specific volume of fluid at inlet to vent = 0.381 ft^3/lb

Before the expansion factor Y and pressure drop ΔP can be found, it must be determined if the flow at the vent exit is sonic. This is done by calculating the ratio $\Delta P/P_1'$ for the system, (where P_1' is the reservoir pressure in absolute units) and comparing this to the critical ratio necessary for sonic flow.

For vent B,

$$P_1' = 115 \text{ psia}$$

$$\frac{\Delta P}{P_1'} = \frac{100}{115} = 0.869$$

From pg. A-22 of Ref. 2, the limiting $\Delta P/P_1'$ for sonic velocity with K = 8.0 and k = 1.4 is 0.762. This is smaller than the ratio of 0.869, and therefore sonic velocity is expected at the exit of the vent.

The actual ΔP can now be calculated using the limiting ratio of 0.762.

$$\frac{\Delta P}{P_1'} = 0.762$$

$$\Delta P = 115\,(0.762) = 87.6 \text{ psi}$$

The expansion factor Y is available directly from the chart on pg. A-22 of Ref. 2.

$$Y = 0.685$$

Then, applying the modified Darcy formula

$$W = 0.525\ (0.685)(6.407)^2 \left(\frac{87.6}{8(0.381)}\right)^{1/2}$$

$$W = 78 \text{ lbs/sec}$$

The same procedure applied to vent A indicates sonic velocity with

$$\Delta P = 87.2 \text{ psi}$$
$$Y = 0.683$$
$$W = 79 \text{ lbs/sec}$$

A flow rate of 80 lbs/sec will be used for both vents A and B in the force calculations. A ΔP of 87 will be used for both vents in the component pressure calculations.

III. Calculation of Pressure Drops in Components

Pressures were calculated for the inlet and outlet of each vent component by beginning with the reservoir pressure and distributing the 85 psi pressure drop according to each component's resistance K. For example, for component 1 of vent B,

$$K = 2.82 \text{ and } \Delta P = \frac{2.82}{8}(87) = 30.7 \text{ psi}$$

across this entrance. Table II shows the pressures as calculated at each point in Fig. 2 for vent A and vent B.

IV. Calculation of Velocities in Components

The velocities were calculated at the entrance and exit of each vent component from the flow rate, and fluid density. The density is known because the pressure and temperature are known at each point. The appropriate equation for velocity is

$\rho VA = W$ where
- V = velocity (in/sec)
- A = component cross sectional area (in^2)
- ρ = weight density of fluid (lb/in^3)
- W = flow rate (lb/sec)

Then,

$$V = \frac{W}{\rho A}$$

Table III shows the velocity as calculated at each point in Fig. 2 for vent A and vent B.

V. Calculation of Inertial Forces on Piping Components

The fluid in each component sees pressure forces from adjacent fluid elements, and a lateral pressure and shear from the vent component. To calculate the forces which the vent component applies to the fluid passing through it, a free body diagram of the fluid alone is drawn, and the principle of linear impulse momentum is applied.

This process is illustrated for the typical elbow of Fig. 3. The external forces F_x and F_y are assumed to act through the fluid centroid, thereby producing no moment on the fluid. Then the linear impulse - momentum principle can be used to solve for F_x and F_y.

In the x-direction,

initial linear momentum + linear impulse = final linear momentum

$$MV_{x1} + \Sigma F_x \cdot t = MV_{x2}$$

where ΣF_x = sum of the forces in the x-direction

M = mass which has its momentum changed in time t.

If t is assumed to be one second for simplicity, then

$$\frac{W_1}{g} V_{x1} + F_1 - F_x = 0$$

and

$$F_x = \frac{W_1}{g} V_{x1} + F_1$$

where

W_1 = Wx1 second, the flow by weight in one second

g = acceleration due to gravity (386.4 in/sec^2)

The force F_y can be calculated in an analogous manner by considering forces and momentum in the y direction.

Once the forces on the fluid elements with the components have been calculated, it is known from equilibrium that the component sees a force from the fluid equal in magnitude and opposite in sign. Table IV lists the forces applied by the fluid on the components of Fig. 2, using the coordinate system shown in that figure.

VI. Finite Element Models of Vents

Finite element models were made of both vent systems. Pipe elements were used on straight runs, and elbow elements were used for the 90° and 45° elbows. These piping elements accept internal pressure and calculate the principle stresses and maximum shear stress resulting from all loadings. In addition, the elbow element has a flexibility which reflects the true behavior of a curved piping component.

The models and constraints are shown in Fig. 4. The vents were each constrained in the following manner:

1. All translations and rotations were constrained where the vent and dewar mate.

2. X and Y translations were constrained where the vents leave the building at the wall. The vents are free to slide in Z.

3. A midpoint of each vent was constrained in vertical (Y direction) translation to prevent bowing under the thermal and force loads.

The maximum stress, deflections and reaction forces in each vent are summarized in Figs. 5 and 6.

The results for both vents indicate that a force of approximately 2350 lbs exists at the connection of the vent to the dewar. To find the net force at this connection it is necessary to consider the forces applied to the dewars by the fluid. Figure 7 shows a free body diagram of the fluid in the dewar of vent B. Although it is not strictly correct to apply the steady state momentum impulse principle to this system, it is assumed that this is sufficiently accurate for a short time interval just after venting starts.

For momentum in the x-direction,

$$MV_{x1} + \Sigma F_x \cdot t = MV_{x2}$$

$$0 + 1475 + F_x = 0.207\ (-3159)$$

$$F_x = -2129 \text{ lbs}$$

$$\dot{M} = \frac{80 \text{ lbs/sec}}{386.4 \text{ in/sec}^2} = 0.207$$

The force which the fluid exerts on the dewar is then 2129 for vent B, and by an analogous derivation, -2129 for vent A. Then the unequilibrated force on the vent dewar connections is 2350-2129 = 221 lbs. This force must be resisted by the dewar itself and any external bracing as might be used. In the actual installation, the vertical run (component 4) is externally braced to the legs of the dewar. The dewar will still take the greater part of the vent reaction at the vent connection, because it is much stiffer than the external bracing. However, the external bracing should prevent the vents from causing harm to personnel or equipment in the unlikely event that the vent connections to the dewars should fail.

The flanges by which the vent connections are made to the dewars must resist the entire 2490 lb force. These flanges are ANSI B16.5 Class 150 and rated at 275 psig. This pressure alone results in a force of 4057 lbs on the 4" flange used at the connection. It is obvious that these flanges are adequate for the worst case force calculated for venting.

The pipe stresses at the vent connections are greater in vent A. This is due to the greater stiffness in the z-direction which this vent has due to the lack of a substantial x-direction run (such as that of component 8 in vent A). The result is that z-displacements produce significant moments on the connection (see Fig. 5 and 6). However, the calculated value of 5242 psi is far below the yield strength of the vent material.

The largest vent stresses occur at the elbow at the building wall. These stresses, approximately 27000 psi, are not quite at the yield strength of the material, and are not unreasonable given the assumptions of this analysis.

Conclusion

The vents for the 30-inch Tohoku Bubble Chamber magnets are adequately sized and sufficiently strong to resist the forces and contractions expected from a "worst case" venting scenario.

References

1. ASM Metals Reference Book, American Society for Metals, 1981.

2. Flow of Fluids through Valves, Fittings, and Pipe, Tech. Paper No. 410, Crane Co., 1978.

Table I
Dimensions and Resistances of Vent Components

Component	Vent B I.D. (in)	Vent B Length (ft)	Vent B Resistance K	Vent A I.D. (in)	Vent A Length (ft)	Vent A Resistance K
1	4.334	2	2.82	4.334	2	2.82
2	*	*	1.4	*	*	1.4
3	6.407	**	0.228	6.407	**	0.228
4	6.407	5.33	0.150	6.407	5.33	0.150
5	6.407	**	0.228	6.407	**	0.228
6	6.407	6.5	0.182	6.407	12.3	0.345
7	6.407	**	0.228	6.407	***	0.163
8	6.407	8	0.225	6.407	3	0.084
9	6.407	***	0.163	6.407	***	0.163
10	6.407	19	0.53	6.407	27	0.758
11	6.407	***	0.163	6.407	***	0.228
12	6.407	11	0.309	6.407	5	1.14
13	6.407	**	0.228	--	--	--
14	6.407	5	1.14	--	--	--
TOTAL			8.00			7.7

* Concentric 6x4 reducer
** Standard 90° elbow
***Standard 45° elbow

NOTE: Resistances of first and last components in each vent include entrance and exit resistances.

Table II
Pressures in Vents

Point	Vent B Pressure (psia)	Vent A Pressure (psia)
1	115	115
2	84	83
3	69	67
4	66	64
5	64	62
6	61	59
7	59	55
8	56	53
9	54	52
10	52	50
11	46	41
12	44	38
13	41	28
14	38	--
15	28	--

Table III
Velocities in Vent

Point	Vent B Velocity (in/sec)	Vent A Velocity (in/sec)
1	3159	3159
2	4228	4593
3	2604	2694
4	2741	2837
5	2837	2941
6	2996	3111
7	3111	3365
8	3298	3508
9	3435	3584
10	3584	3743
11	4104	4653
12	4311	5055
13	4653	7001
14	5055	- -
15	7001	- -

Table IV
Forces on Vent Components (lbs)

Component	Vent B FX	Vent B FY	Vent B FZ	Vent A FX	Vent A FY	Vent A FZ
1	-236	0	0	175	0	0
2	647	0	0	-540	0	0
3	-2761	2692	0	2714	2648	0
4	0	-44	0	0	-43	0
5	0	-2648	2584	0	-2604	2544
6	0	0	-40	0	0	-76
7	-2485	0	-2544	1720	0	-747
8	36	0	0	-12	0	-12
9	742	0	1707	-1707	0	677
10	61	0	-61	0	0	-102
11	1647	0	661	0	-2270	-2283
12	0	0	-26	0	-81	0
13	0	-2270	-2283	--	--	--
14	0	-81	0	--	--	--

Appendix 1
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March 7, 1985

Elbow at Building Wall

The maximum stress intensity in the piping is 27,200 psi and is found at both elbows at the building wall. The pressure at this location is ~40 psi. The analysis in this document assumed schedule 5 pipe (0.109" wall). Actually schedule 10 pipe (0.134" wall) was installed. This will change the pressure drops very slightly but can also be expected to significantly reduce stress in the piping. Section VIII Division 2 will be used to compare the stress levels.

For 304 welded stainless steel, S_m = 17 ksi with Fermilab's derating factor is reduced to 17,000 x 0.8 = 13,600 psi. The following estimates are made.

$$\sigma_\theta = \frac{Pr}{t} = 1180 \text{ psi from internal pressure}$$

$$\sigma_{axial} = \frac{\text{Force}}{\text{Area}} = \frac{2280}{2.194} = 1040 \text{ psi from inertial force}$$

$$\sigma_r = 40 \text{ psi}$$

It is clear that virtually the entire stress arises from bending.

$$P_m \text{ (S.I.)} = 1180 + 1040 = 2220 \leqslant S_m = 13,600 \text{ psi}$$

The bending stress in the elbow must be classified as primary bending since this stress resists the fluid forces and is not self limiting.

$$P_m + P_b \text{ (S.I.)} = 27,000 > 1.5\ S_m = 20,400$$

However, the pipe and elbows are schedule 10. Since bending stress is proportional to $1/t^2$, the true value of $P_m + P_b$ (S.I.) = 27,000 x $(0.109/0.134)^2$ = 17,900 psi. Furthermore, the elbow is actually forged and not welded so a higher value of S_m might have been chosen depending upon where the peak stress value actually occurred in the elbow.

Conclusion

The elbow at the building wall has satisfactory strength.

Welded Pipe Connection to Dewar

This welded connection is more properly treated in the LHe dewar document but will be considered here to eliminate paper work. For the internal pressure load this weld was already judged to have satisfactory strength.

The reaction forces and moments at the welded connection to dewar A are given in Fig. 6 of this report. The stress intensity is calculated by ANSYS to be 5242 psi. The peak stress value is for a 4" schedule 5 pipe (0.083" wall) and not for the weld. The LHe dewar report assumes this weld to be a 1/16" single fillet weld with a 0.044" throat. The true measured value is $t_{leg} > 3/16" = 0.1875"$. The throat then is 0.1875 x 0.707 = 0.133". This is greater than the assumed thickness of the pipe, and the peak stress intensity is less than 5240 psi.

Conclusion

The welded vent pipe/dewar connection has a much larger than required safety factor.

Fig 1 30' BC Helium Dewar Vent Piping
(not to scale)

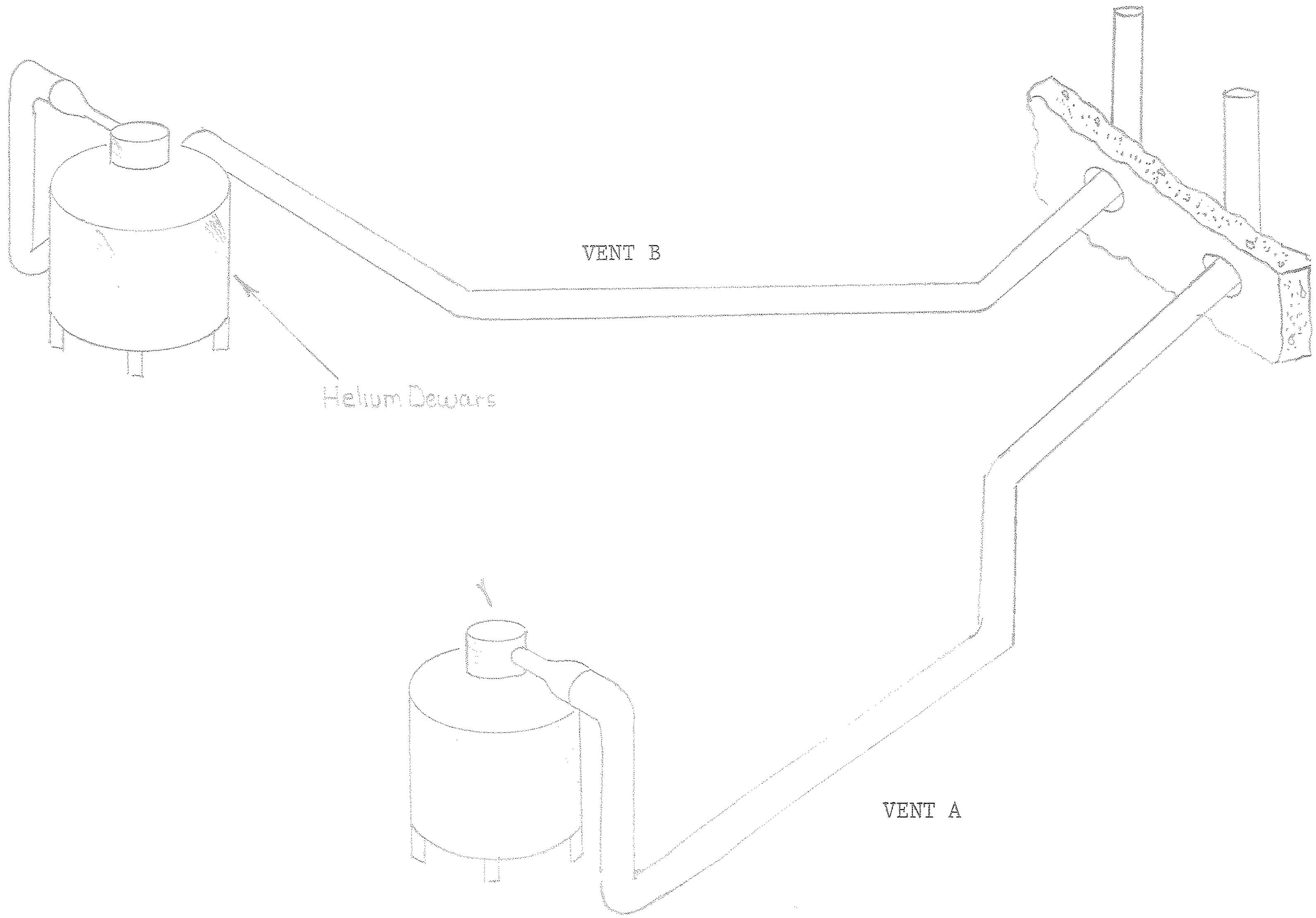
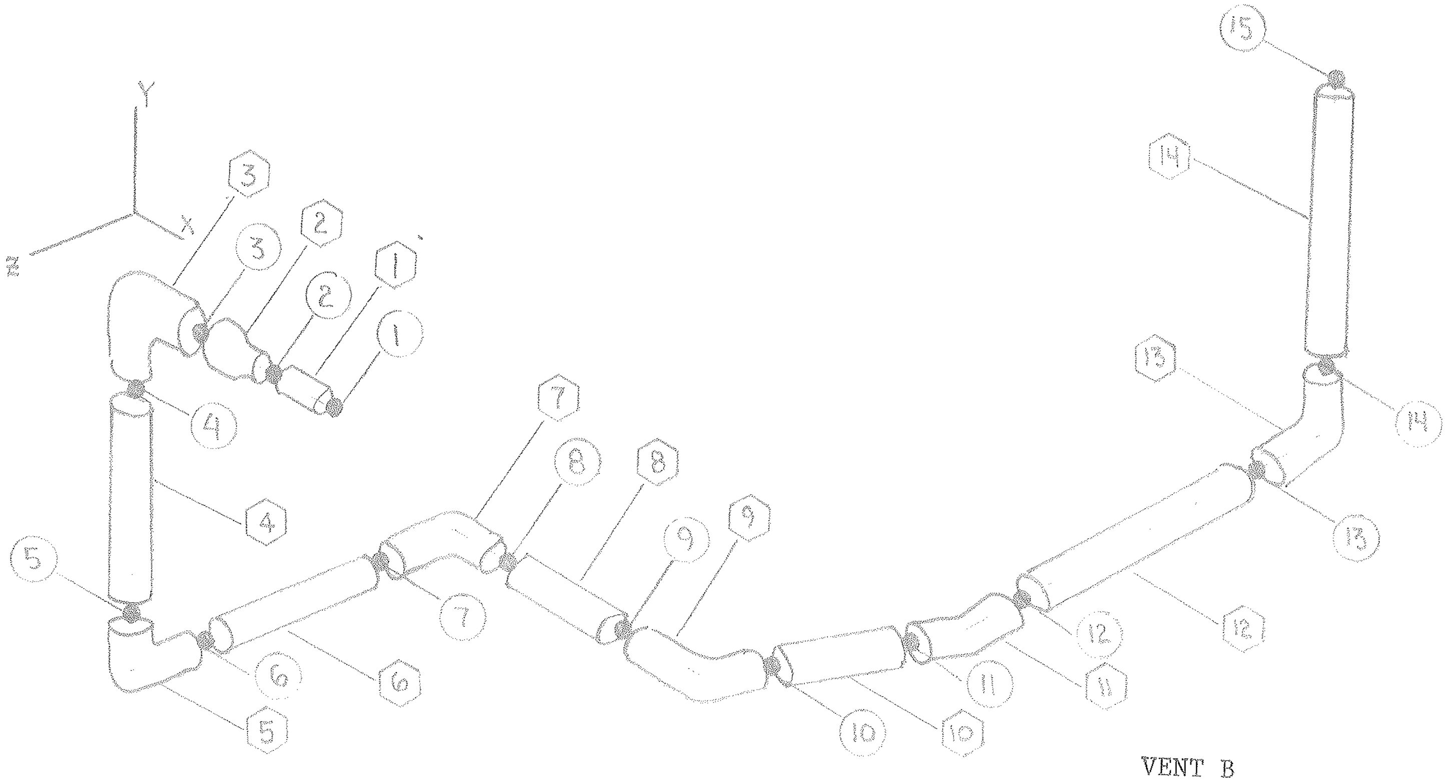
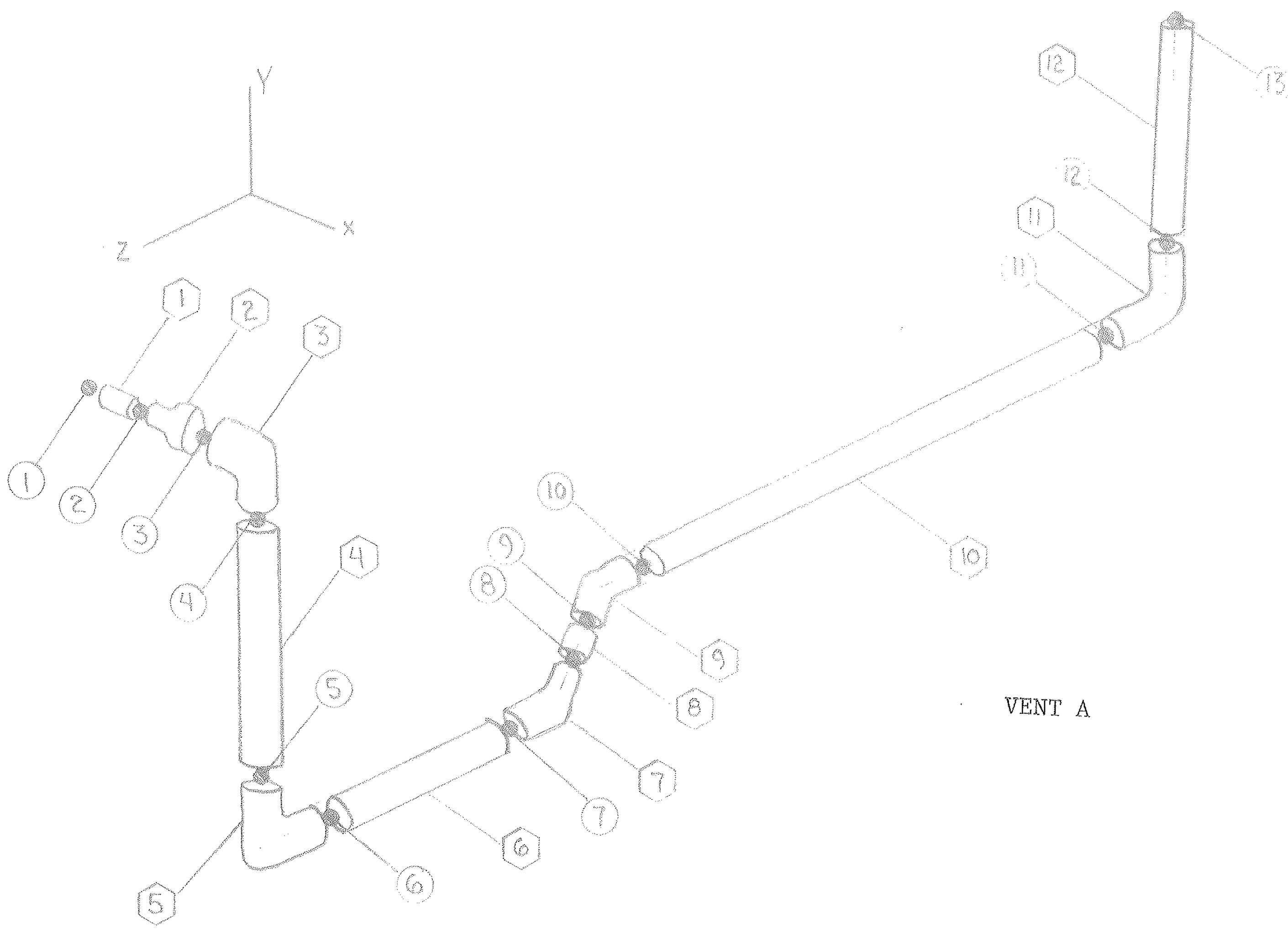
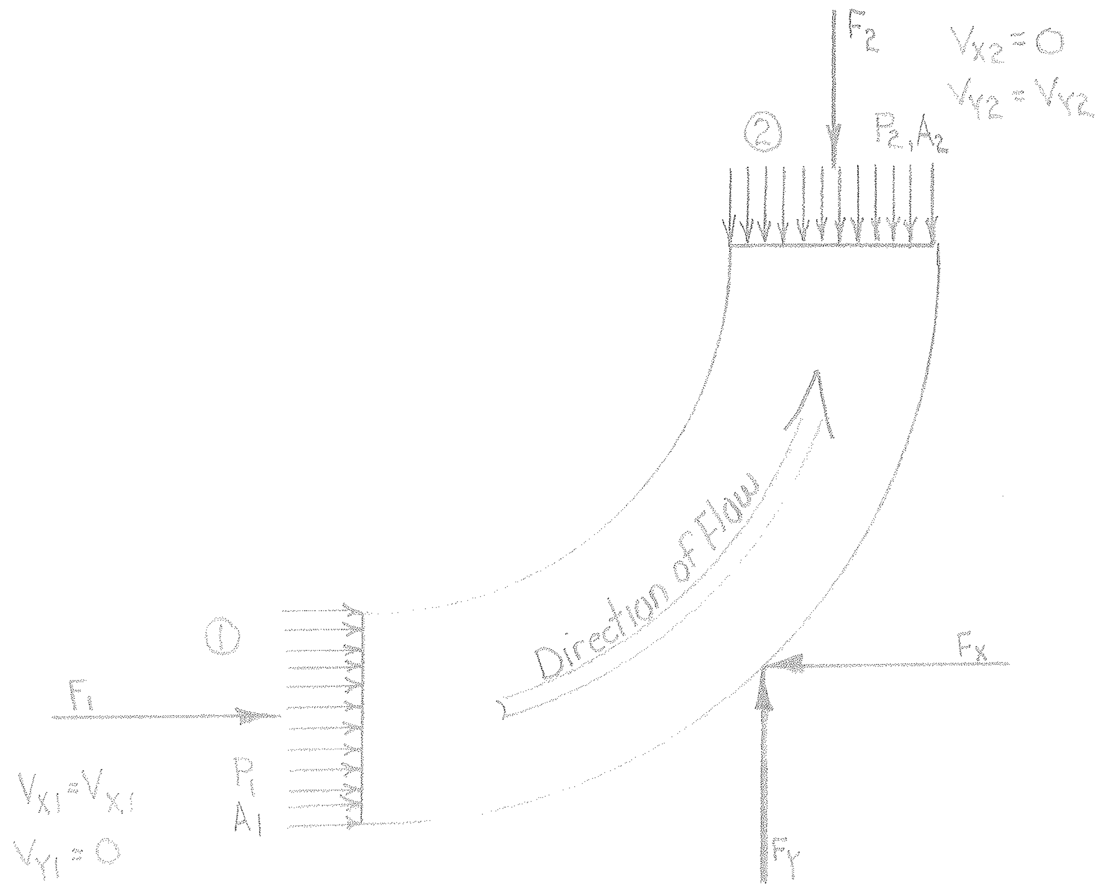


Fig 2. Vent Components







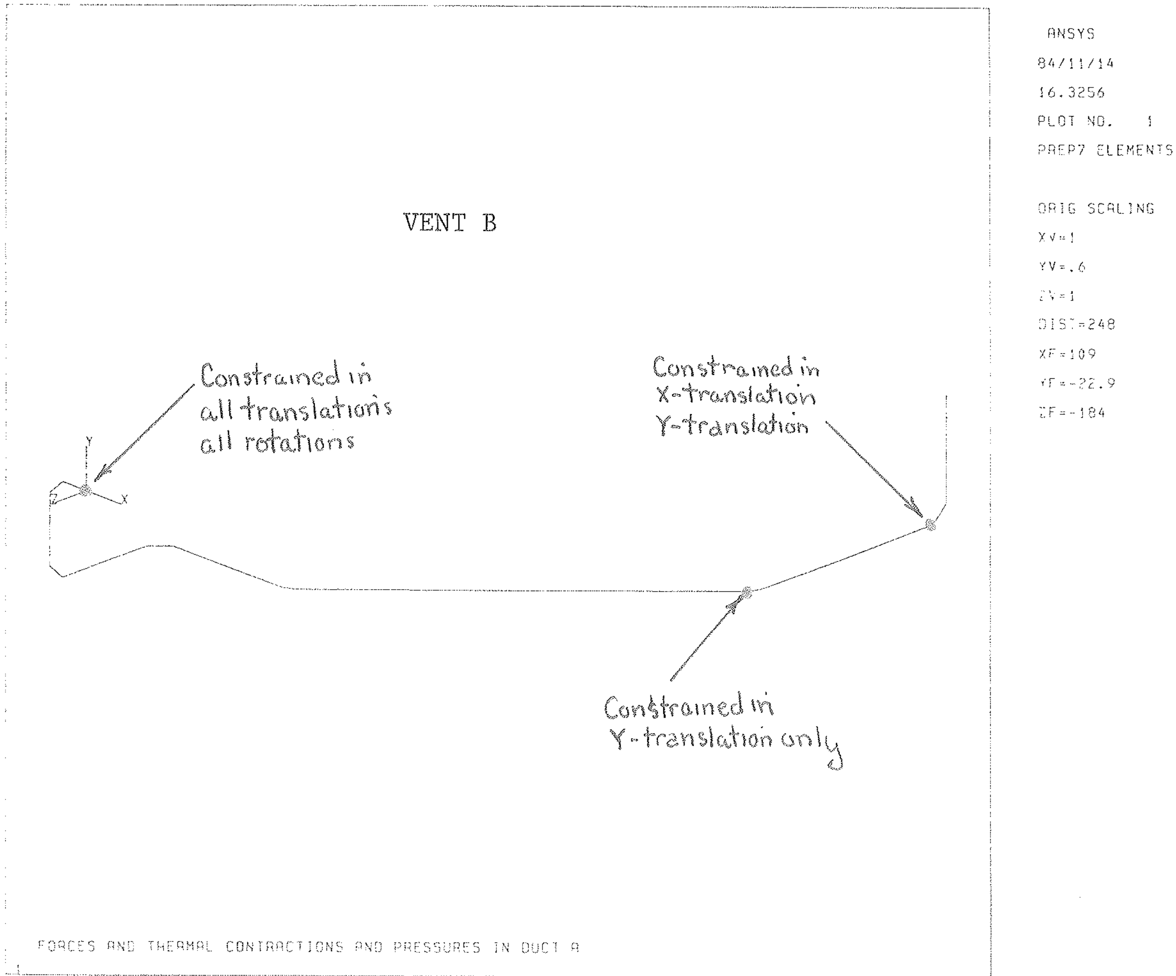
F_i = pressure force = $P_i A_i$

V_{xi} = component of velocity in x-direction

V_{yi} = component of velocity in y-direction

F_x, F_y = force applied to fluid by elbow

Fig 3. Free Body Diagram of Fluid in Elbow



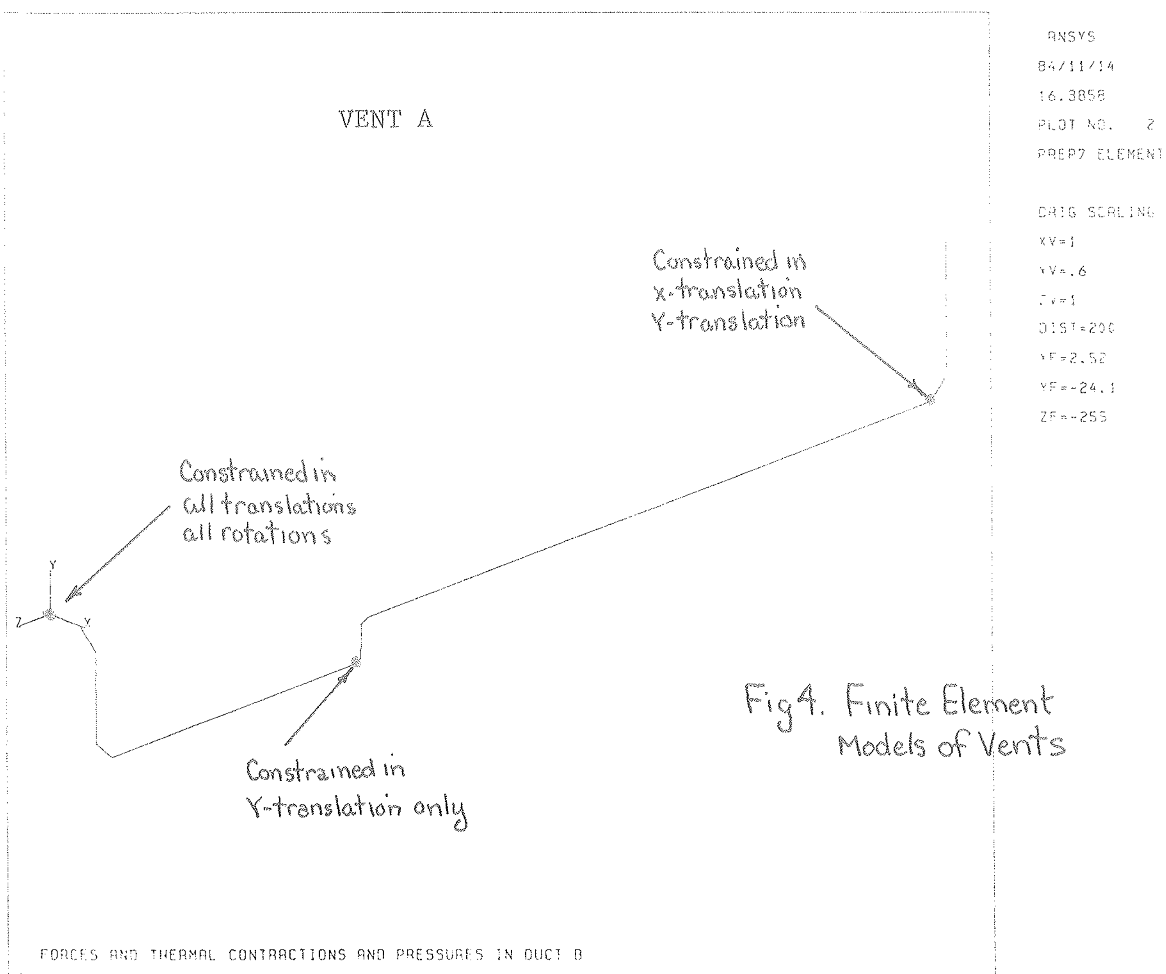


Fig 4. Finite Element Models of Vents

Location	Reaction Forces			Stress Intensity	Displacements		
	FX	FY	FZ	(PSI)	X	Y	Z
	(lbs)	(lbs)	(lbs)		(in)	(in)	(in)
A	2343	0	0	4895	0	0	0
B	0	0	0	3845	.15	.05	.34
C	0	-173	0	4314	-.38	0	1.26
D	0	2523	0	27200	0	0	1.72
E	0	0	0	0	.07	-.30	1.05

Moments @ A

MX	MY	MZ
(in-lbs)	(in-lbs)	(in-lbs)
-621	1728	-309

FORCES AND THERMAL CONTRACTIONS AND PRESSURES IN DUCT A

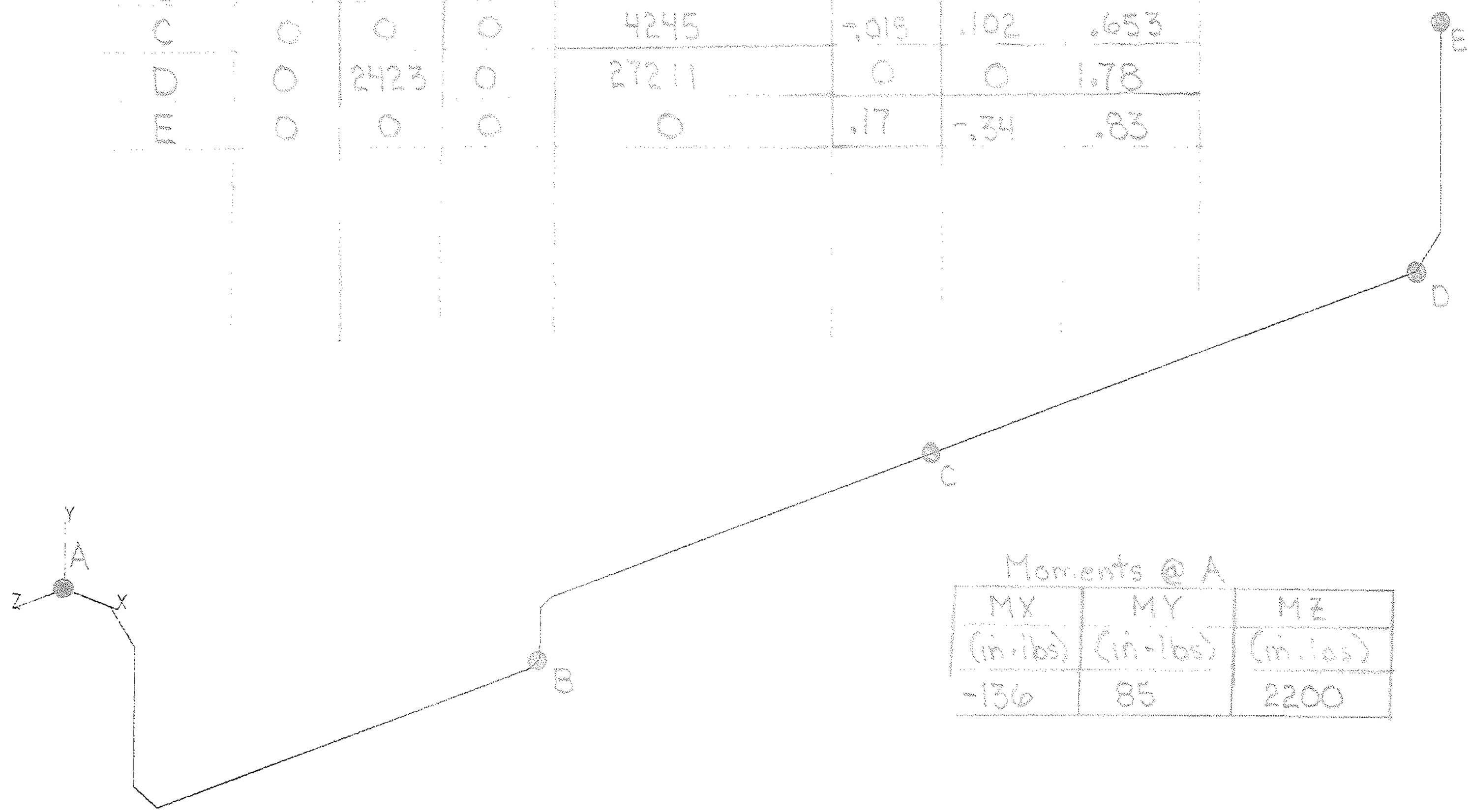
ANSYS
84/11/14
16.3256
PLOT NO. 1
PREP7 ELEMENTS

ORIG SCALING
XV=1
YV=.6
ZV=1
DIST=248
XF=109
YF=-22.9
ZF=-184

Fig 5. Summary of Finite Element Results for

VENT B

Location	Reaction Forces FX (lbs)	Reaction Forces FY (lbs)	Reaction Forces FZ (lbs)	Stress Intensity (PSI)	Displacements X (in)	Displacements Y (in)	Displacements Z (in)
A	-2351	24	0	5242	0	0	0
B	0	-100	0	8307	-.12	0	.54
C	0	0	0	4245	-.019	.102	.653
D	0	2423	0	27211	0	0	1.78
E	0	0	0	0	.17	-.34	.83



Moments @ A

MX (in-lbs)	MY (in-lbs)	MZ (in-lbs)
-136	85	2200

ANSYS
84/11/14
16.3858
PLOT NO. 2
PREP7 ELEMENTS

ORIG SCALING
XV=1
YV=.6
ZV=1
DIST=200
XF=2.52
YF=-24.1
ZF=-255

FORCES AND THERMAL CONTRACTIONS AND PRESSURES IN DUCT B

Fig 6. Summary of Finite Element Results for VENT A

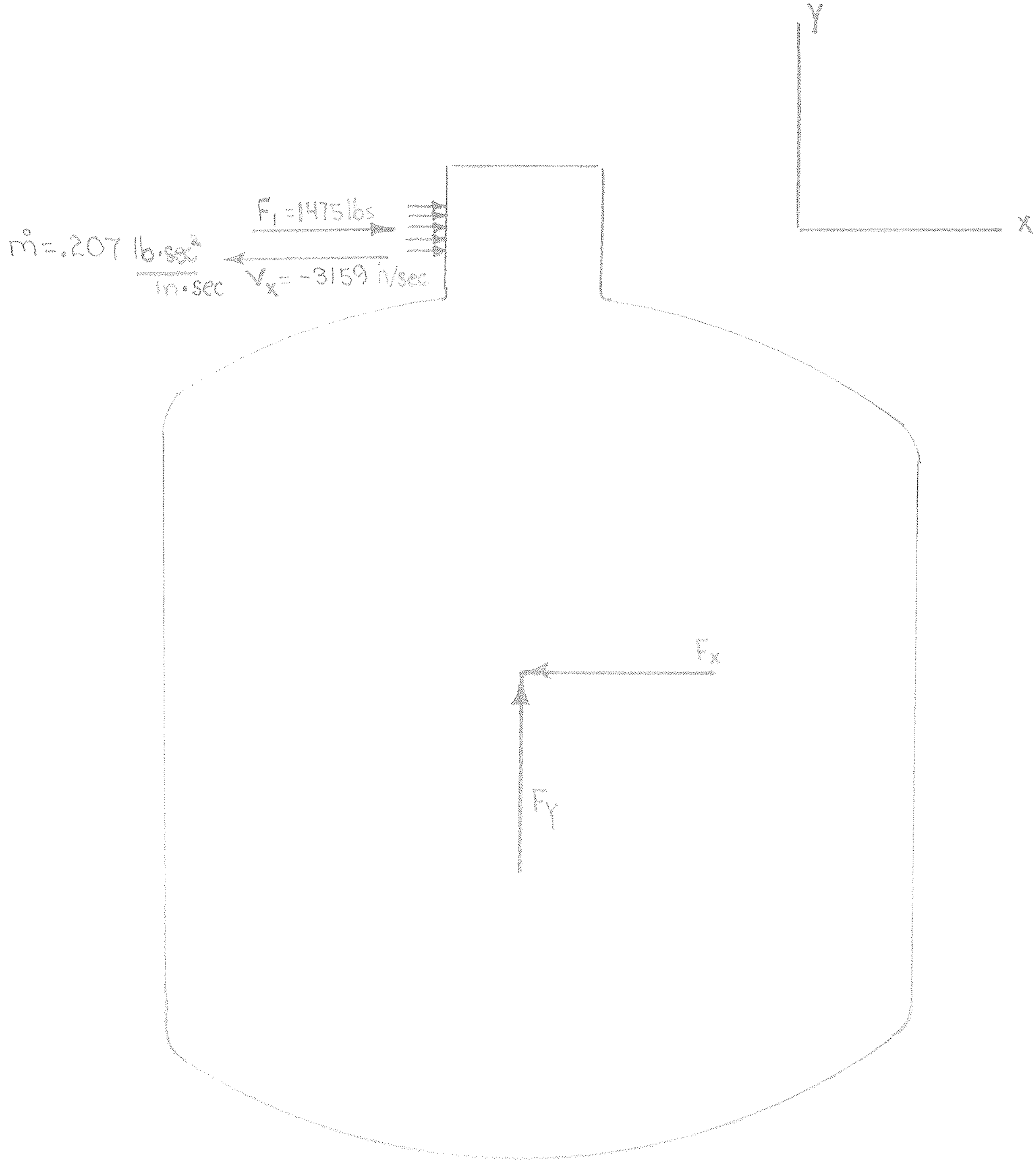


Fig 7. Free Body Diagram of Fluid in Dewar